Those exercises marked as home exercises must be returned latest Wed 1.11, 14 o'clock into the box in G-wing marked S-38.118 Exercises 1 - 3 are demonstrated (in Finnish) during exercise class by the assistant. Exercises 4 - 6 are home exercises. Each home exercise is worth 0-2/points. You may receive help in solving the home exercises during exercise class after the assistant has explained exercises 1 - 3.

1. Little's formula (Demo)

   Let us observe the system at hand as a black box. We know that customers arrive to the system with average intensity (speed) λ. Let us denote the average time that a customer spends in the system by $\bar{T}$ and the average number of customers inside the system by $\bar{N}$. Little's formula states that

   $$\bar{N} = \lambda \bar{T}.$$

   Prove the formula using the following setting:

   Assume that each customer i is billed the amount M_i. The magnitude of M_i is proportional to the time spent by the customer in the system, $M_i \propto T_i$ mk/min, in other words the customer is billed for the time spent in the system. What is the average income rate of the system (the income gathered during a time period/length of time period)? Give an interpretation of the income rate, if each customer i pays a fee $M_i = T_i$.

2. Basics on loss networks: (Demo)

   We have a loss system with three servers (channels). Customers arrive with intensity $\lambda = 1/\text{s}$ and the average service time is 2s.

   a) Calculate the time blocking probability of the system.

   b) How many customers are on average blocked during an hour?

   c) What are the offered, carried, and blocked traffic intensities of the system?

3. Study the maximum throughput of the ALOHA multiple access technique in computer Local Area Networks (LAN).

   The computers (stations) that are connected to the LAN send packets to each other. When sending packets, the stations compete for the joint transmission medium. The resource can only be used by one station at a time. The dynamic channel assignment is made in a fully distributed manner by the stations themselves.

We study the ALOHA random access technique. Stations send each other packets independently with no prior actions to avoid collision. Assume that the stations generate fixed length packets (the time to send a packet is T) according to a Poisson process with intensity ν. The stability requirement is $\nu < 1/T$. Two packets collide if their interarrival time is $< T$. Collided packets are retransmitted after a random interval as long as the transmission has succeeded. We approximate the aggregate packet stream by a Poisson process, with intensity $\lambda > \nu$. What is the maximum throughput of the ALOHA mechanism?

4. Performance measures for queuing systems (homework 2p.)

   The $M/M/1$ queue is used to model situations where customers arrive, with exponentially distributed interarrival times (parameter λ), to a system with one server. The customers service time is exponentially distributed (parameter μ) and the number of waiting places is infinite. Let us denote the number of customers in the system by the random variable X. It can be shown that, in equilibrium, the state probabilities of the system $\pi_i = P(X = i)$, i.e. the probabilities of being in state i, are from the geometrical distribution.

$$\pi_i = \pi_0 \Big(\frac{\lambda}{\mu}\Big)^i = (1-\rho)\rho^i$$

   a) Calculate the expected value of the customers (average number of customers) in the $M/M/1$ queue.

   b) Using Little's Formula, calculate the average time spent by the customer in the system.

   c) What is the average waiting time of a customer (waiting time is the time spent in the buffer before being serviced)?

      Draw a picture to illustrate how the arrival intensity λ affects the waiting time. Assume that $\mu = 1$.

5. Dimensioning (homework 2p.)

   Consider a mail order company, with 3 persons answering order calls. The calls arrive as from a Poisson process with intensity 1/min and the average duration of a call is 2 min.

   a) Calculate the probability that an arriving call is blocked, assuming that blocked calls are not renewed.

b) Should a fourth person be hired to answer the calls, if the cost of an extra person is 100 mk/h and the revenue per subscription is on average 20 mk?

6. Visualizing the traffic process (homework 2p.)

   Consider telephone traffic in a link between two telephone exchanges. During the time interval $[0, T]$, where $T = 16$ (time units), there are 7 new calls arriving at the moments

   - 1, 2, 4, 5, 6, 9 and 12 (time units).

   The holding times (if the calls are not blocked) are respectively

   - 9, 5, 4, 1, 7, 2 and 6 (time units).

   The link capacity is assumed to be $n = 3$ channels. Assume further that, at time $t = 0$, the system is empty (that is, all the three channels are free).

   Draw a picture describing the call arrival times, channel occupation (for each channel separately), and the number of occupied channels as a function of time $t \in [0, T]$. How many of the calls offered are blocked? What is the call blocking probability? What is the proportion of time that the system is in the blocking state? What is the total traffic volume in this time interval $[0, T]$?