HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 5**
Laboratory of Telecommunications Technology | 18.10.2000
S-38.145 Introduction to Teletraffic Theory, Fall 2000 | Aalto/Nyberg

Note: Problem 3 is a homework exercise. Deliver your answer sheet (labelled with your student id, name, and signature) into the mail box of the course, or directly to the course assistant *before* the next exercise class on 25 October.

1. Let X_1 and X_2 be two independent and identically distributed (IID) random variables, $X_i \sim \text{Exp}(\lambda)$, $\lambda > 0$ $(i = 1, 2)$. Let Z denote their sum,

$$Z = X_1 + X_2.$$

   a) Determine the value set, probability density function (pdf), cumulative distribution function (cdf), mean value $E[Z]$, and variance $D^2[Z]$ of random variable Z.

   b) Assume then that $\lambda = 1$, and calculate the standard deviation $D[Z]$ and the coefficient of variation $C[Z]$ of Z.

   (*Tip:* Derive first cdf starting from the equation $P\{Z \le z\} = \int_0^z P\{X_1 \in dx, X_2 \le z-x\}$ and utilizing the independence.)

2. Consider a pure waiting system with two parallel servers. Service times are assumed to be independent and exponentially distributed with mean $1/\mu > 0$ (seconds).

   a) Assume first that there is only one customer in the system at time 0 when a new customer arrives. Thus, the service of the "new" customer starts immediately. Assume further that the "old" customer has already been served for x (seconds). What is the probability that the "new" customer leaves the system last?

   b) Assume then that there are already two customers in the system at time 0 when a new customer arrives. Thus, the "new" customer has to wait until one of the "old" customers leaves the system. Assume further that both of the "old" customer have already been served for x (seconds). What is the probability that the "new" customer leaves the system last?

   (*Tip:* Utilize the memoryless property of exponential distribution.)

3. *Homework exercise* (deadline 25 October at 9 o'clock): Consider still a pure waiting system with two parallel servers. As in the previous problem, service times are assumed to be independent and exponentially distributed with mean $1/\mu > 0$ (seconds).

   Assume that the system is empty at time 0 when three new customers arrive together. Assume further that no new customers arrive after them. Let T denote the time until the system is empty again. What is the mean value of this random variable T?