HELSINKI UNIVERSITY OF TECHNOLOGY	**Exercise 6**
Laboratory of Telecommunications Technology	25.10.2000
S-38.145 Introduction to Teletraffic Theory, Fall 2000	Aalto/Nyberg

Note: Problem 3 is a homework exercise. Deliver your answer sheet (labelled with your student id, name, and signature) into the mail box of the course, or directly to the course assistant *before* the next exercise class on 1 November.

1. Let τ_n be a Poisson process with intensity $\lambda > 0$ and $\tau_0 = 0$. According to Definition 2 in slide 21 of lecture 6, the interarrival times $\tau_n - \tau_{n-1}$ are independently and exponentially distributed with mean $1/\lambda$. Let $A(t)$ denote the corresponding counter process so that

$$A(t) = \max\{n = 0, 1, 2, \ldots \mid \tau_n \le t\}, \quad t \ge 0.$$

Prove (without help of Definition 3) that

a) $P\{A(t) = 0\} = e^{-\lambda t}$

b) $P\{A(t) = 1\} = \lambda t e^{-\lambda t}$.

(*Tip:* Calculate first $P\{A(t) \ge 0\}$, $P\{A(t) \ge 1\}$, and $P\{A(t) \ge 2\}$. Utilize here exercise 5.1.)

2. Consider the following Markov processes with state space $S = \{0, 1, 2\}$. The processes are defined by giving all the state transition rates q_{ij}, $i \ne j$, in the following table ("0" means that $q_{ij} = 0$, and "+" means that $q_{ij} > 0$).

(i,j)	(0,1)	(0,2)	(1,0)	(1,2)	(2,0)	(2,1)
a)	0	+	+	0	0	+
b)	+	+	+	0	0	+
c)	+	0	+	+	0	+
d)	0	+	0	0	0	+
e)	+	+	+	+	+	+
f)	+	+	+	+	+	+

Draw the state transition diagram for each process. Which processes are irreducible?

3. *Homework exercise* (deadline 1 November at 9 o'clock): Consider still the Markov processes defined in the previous problem. We refine their definitions by giving more explicitly the state transition rates q_{ij} in the following table:

(i,j)	(0,1)	(0,2)	(1,0)	(1,2)	(2,0)	(2,1)
a)	0	10	1	0	0	1
b)	10	10	1	0	0	1
c)	10	0	1	1	0	1
d)	0	10	0	0	0	1
e)	10	1	10	1	1	1
f)	10	1	1	10	10	1

a) Which processes are irreducible *and* positively recurrent (that is: Which of them have an equilibrium distribution)? b) Determine the equilibrium distribution (whenever it exists). c) Which processes are reversible (that is: Which of them satisfy all the local balance equations)?