Note: Problem 3 is a homework exercise. Deliver your answer sheet (labelled with your student id, name, and signature) into the mail box of the course, or directly to the course assistant *before* the next exercise class on 8 November.

1. Consider a link in a circuit switched (trunk) network. Denote by n the number of parallel channels in the link. Assume that traffic sources generate new connections according to a Poisson process (each connection occupies one channel). The mean interarrival time between new connection requests is denoted by t, and the mean connection holding time by h. Use the Erlang model to analyze this system. Calculate the time blocking and the call blocking in the following cases:

   (a) $n = 2$, $t = 3$ min, and $h = 3$ min,

   (b) $n = 2$, $t = 4$ min, and $h = 3$ min.

   What are the traffic offered and the traffic carried in these cases?

2. Consider a concentrator in a circuit switched (access) network (see slides 35–37 of lecture 2). Traffic (= connections) from n 1-channel access links is concentrated on a single m-channel link connected to the nearest network node, where $m < n$. Traffic on each access link is generated by an on-off type source (one source per link). The sources are assumed to be independent and identical. The mean idle period is denoted by t, and the mean active period by h. Use the Engset model to analyze this system. Calculate the time blocking and the call blocking in the following cases:

   (a) $n = 4$, $m = 2$, $t = 9$ min, and $h = 3$ min,

   (b) $n = 3$, $m = 2$, $t = 9$ min, and $h = 3$ min.

3. *Homework exercise* (deadline 8 November at 9 o'clock): Consider the Engset model. Let π_i denote the equilibrium probability for state i. In slide 29 of lecture 7, the following result was derived:

   $$\pi_i = \binom{k}{i} \left(\frac{\nu}{\mu}\right)^i \pi_0, \qquad i = 0, 1, 2, \ldots, n.$$

   Let then π_i^* denote the probability that there are i active customers when an idle customer becomes active. In slide 33 of lecture 7, the following result was derived:

   $$\pi_i^* = G^{-1}(k - i)\pi_i, \qquad i = 0, 1, 2, \ldots, n,$$

   where $G = \sum_{j=0}^{n}(k - j)\pi_j$. Now prove that

   $$\pi_i^* = \binom{k-1}{i} \left(\frac{\nu}{\mu}\right)^i \pi_0^*, \qquad i = 0, 1, 2, \ldots, n,$$

   where

   $$\pi_0^* = \left(\sum_{j=0}^{n} \binom{k-1}{j} \left(\frac{\nu}{\mu}\right)^j\right)^{-1}.$$