HELSINKI UNIVERSITY OF TECHNOLOGY **Exercise 9**
Laboratory of Telecommunications Technology 15.11.2000
S-38.145 Introduction to Teletraffic Theory, Fall 2000 Aalto/Nyberg

Note: Problem 3 is a homework exercise. Deliver your answer sheet (labelled with your student id, name, and signature) into the mail box of the course, or directly to the course assistant *before* the next exercise class on 22 November.

1. Simulate (according to the discrete event simulation principles presented in the lectures) the evolution of the queue length process $Q(t)$ in an M/M/1-FIFO queue during the interval $[0, T]$ assuming that the system is empty in the beginning ($Q(0) = 0$). Let $\lambda = 1/2$, $\mu = 1$, and $T = 1000$. Make $n = 100$ independent simulation runs. In each simulation run, calculate the mean queue length X from the equation

   $$X = \frac{1}{T}\int_0^T Q(t)dt.$$

   By this way, you get n IID samples $X_1, X_2, \ldots, X_n$ of the mean queue length in this interval. Calculate and plot the sample average $\bar{X}_m$,

   $$\bar{X}_m = \frac{1}{m}\sum_{i=1}^{m} X_i,$$

   for $m = 1, 2, \ldots, n$. Furthermore, calculate and plot the square root of the sample variance S_m,

   $$S_m = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(X_i - \bar{X}_m)^2},$$

   for $m = 2, 3, \ldots, n$. Finally, calculate and plot the confidence interval for the sample average $\bar{X}_m$ at confidence level 95% as $m = 2, 3, \ldots, n$.

   For calculations two approximations can be made: (i) You can assume that the samples are from a normal distribution. (ii) You can approximate the Student($m-1$) distribution by the standard normal N(0, 1) distribution for any m (which yields a bit too narrow confidence intervals for small values of m).

2. Consider still the simulation problem presented above. Now let $T = 10000$ and $n = 1$ so that there is just one long simulation run. Let $X(t)$ denote the mean queue length in the interval $[0, t]$,

   $$X(t) = \frac{1}{t}\int_0^t Q(s)ds.$$

   Calculate and plot $X(t)$ for $t = 100, 200, \ldots, 10000$. Compare these values with the theoretical mean queue length (based on the equilibrium distribution derived in the lectures).

3. *Homework exercise* (deadline 22 November at 9 o'clock): Consider still the simulation problem presented in problem 1 (with $T = 1000$). Let us assume that the desired accuracy of the simulation result is expressed as follows: the width of the confidence interval should be at most $2y$, where $y = 0.01$. By utilizing the results of problem 1, estimate how many independent simulation runs should be done in order to achieve the desired accuracy.

   (*Tip*: The width of the confidence interval is $2y$, when the interval is $(\bar{X}_n - y, \bar{X}_n + y)$.)