# 3. Modelling of telecommunication systems (part 2)



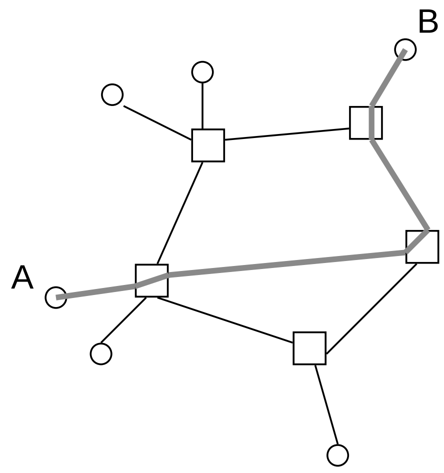


## Contents

- Circuit switched network modelled as a loss network
- Packet switched network modelled as a queueing network

# Teletraffic model of a circuit switched network (1)

- Consider a circuit switched network
  - e.g. a telephone network
- Traffic:
  - telephone calls
  - each (carried) call occupies one channel on each link among its route
- System:
  - telephone machines (terminals)
  - exchanges (network nodes)
  - access links (from terminals to exchanges)
  - trunks (between exchanges)

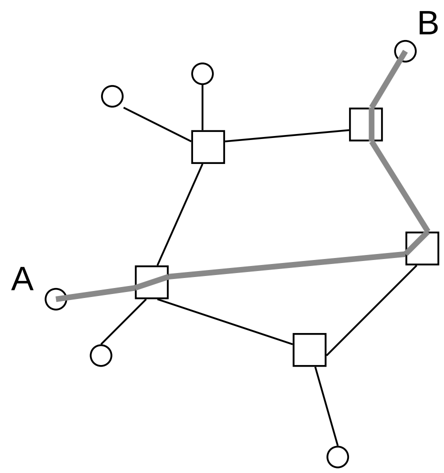






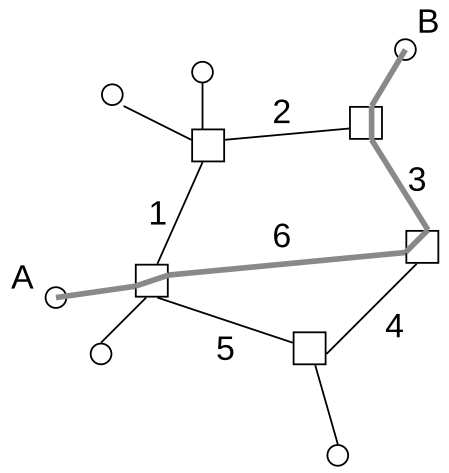

# Teletraffic model of a circuit switched network (2)

- Quality of service:
  - described by the **end-to-end call blocking probability** (prob. that a desired connection cannot be set up due to congestion along the route of the connection)
- In our model we assume that
  - the network nodes and the whole access network are non-blocking
- Thus, a call is blocked
  - if and only if all channels are occupied in any trunk network link along the route of that call



# Links j = 1,...,J

- In our model,
  - all links are **two-way** (why?)
- We index the links in the trunk network by
  - $j = 1,\ldots,J$
- In the example on the right:
  - $J = 6$
- Let
  - $n_j$ = nr of channels in link $j$ (that is: the link capacity)
  - $\mathbf{n} = (n_1,\ldots,n_J)$
- Each link is modelled as a
  - **pure loss system**

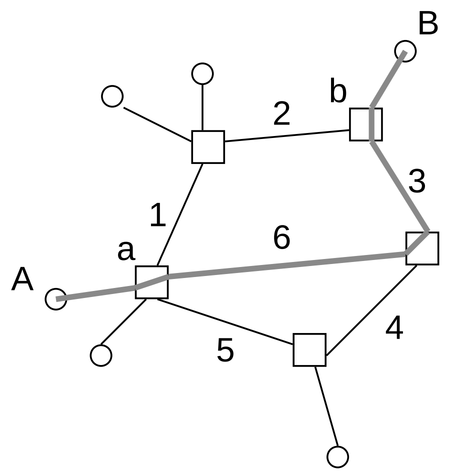






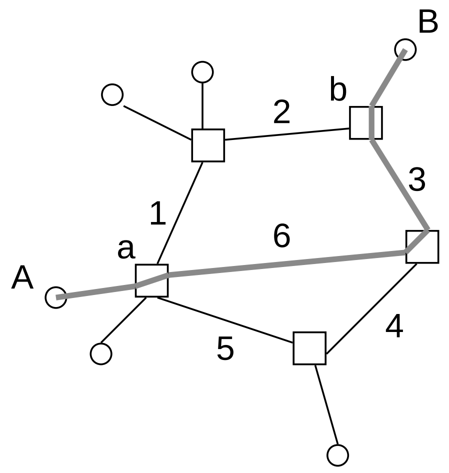

# Routes r = 1,...,R

- We define a **route** as a
  - set of (two-way) links connecting two network nodes
- We index the routes by
  - $r = 1,\ldots,R$
- In the example on the right:
  - there are three routes **between** nodes a and b: {1,2}, {6,3}, {5,4,3}
  - $R = 12 + 10 + 7 + 3 = 32$
- Let
  - $d_{jr} = 1$ if link $j$ belongs to route $r$
  - $d_{jr} = 0$ otherwise
  - $\mathbf{D} = (d_{jr} \mid j = 1,\ldots,J;\ r = 1,\ldots,R)$



# Loss network model

- Note:
  - End-to-end call blocking prob. is equal for all the connections following the same route
- Thus,
  - the traffic class of a connection is determined by the route $r$ that the connection follows
- Let
  - $x_r$ = number of active connections following route $r$
  - $\mathbf{x} = (x_1,\ldots,x_R)$
- Vector $\mathbf{x}$ is called
  - the **state** of the system








# State space S

- The number of active connections $x_r$ for any traffic class $r$ is limited by the link capacities $n_j$ along the corresponding route $r$ :

$$\sum_{r=1}^{R} d_{jr} x_r \leq n_j \quad \text{for all } j$$

- The same in vector form:

$$\mathbf{D} \cdot \mathbf{x} \leq \mathbf{n}$$

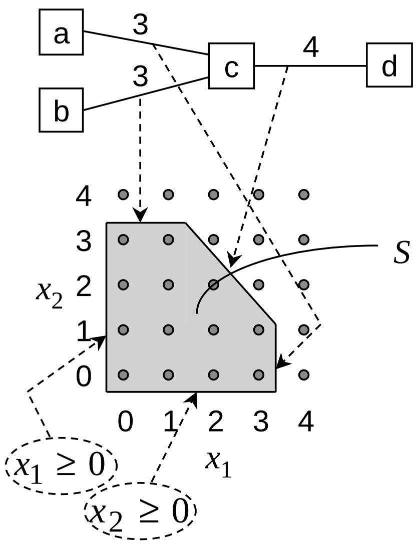
- Thus, the **state space** $S$ (that is: the set of **admissible** states) is

$$S = \{\mathbf{x} \geq 0 \mid \mathbf{D} \cdot \mathbf{x} \leq \mathbf{n}\}$$

- Note that, due to finite link capacities, set $S$ is **finite**

# Example

- 3 links with capacities:
  - link a-c: 3 channels
  - link b-c: 3 channels
  - link c-d: 4 channels
- 2 routes:
  - route a-c-d
  - route b-c-d
  - The other 4 routes (which?) are ignored in this model
- State space:
  - $S = \{(0,0),(0,1),(0,2),(0,3),$ $(1,0),(1,1),(1,2),(1,3),$ $(2,0),(2,1),(2,2),$ $(3,0),(3,1)\}$








# Set $S_r$ of non-blocking states for class r

- Consider
  - an arriving call belonging to class *r* (that is: following route *r*)
- It will **not** be blocked by link *j* belonging to route *r*
  - if there is at least one free channel on link *j*:

$$\sum_{r'=1}^{R} d_{jr'} x_{r'} \leq n_j - 1 \quad \text{for all } j \in r$$

- The same in vector form ($\mathbf{e}_r$ being here the unit vector in direction *r*):

$$\mathbf{D} \cdot (\mathbf{x} + \mathbf{e}_r) \leq \mathbf{n}$$

- The set $S_r$ of **non-blocking** states is thus

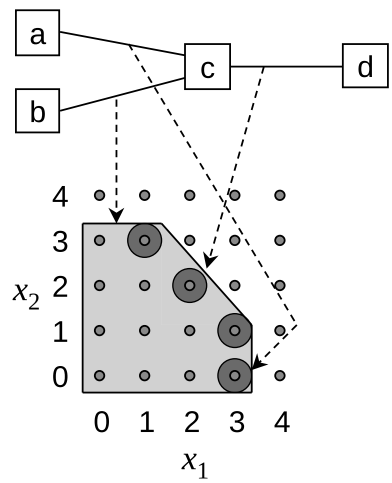
$$S_r = \{\mathbf{x} \geq 0 \mid \mathbf{D} \cdot (\mathbf{x} + \mathbf{e}_r) \leq \mathbf{n}\}$$

# Set $S_r^B$ of blocking states for class r

- The set $S_r^B$ of **blocking** states for class *r* is clearly:

$$S_r^B = S \setminus S_r$$

- Summary:
  - an arriving call of class *r* is blocked (and lost) if and only if the state *x* of the system belongs to set $S_r^B$
- Example (continued):
  - The blocking states $S_1^B$ for connections of class 1 (using route a-c-d) are circulated in the figure
  - $S_1^B$ = { (1,3),(2,2),(3,0),(3,1)}








# Stationary state probabilities (1)

- Assume that
  - new connection requests belonging to traffic class *r* arrive (independently) according to a Poisson process with intensity $\lambda_r$
  - call holding times independently and identically distributed with mean $h$
- Denote
  - $a_r = \lambda_r h$ (traffic intensity for class *r*)

# Stationary state probabilities (2)

- Then it is possible to show that
  - the stationary state probability $\pi(\mathbf{x})$ for any state $\mathbf{x} \in S$ is as follows:

$$\pi(\mathbf{x}) = G^{-1} \cdot \prod_{r=1}^{R} f_r(x_r)$$

where $G$ is a normalizing constant:

$$G = \sum_{\mathbf{x} \in S} \prod_{r=1}^{R} f_r(x_r)$$

and the functions $f_r(x_r)$ are defined as follows:

$$f_r(x_r) = \frac{a_r^{x_r}}{x_r!}$$





# Stationary state probabilities (3)

- Probability $\pi(\mathbf{x})$ is said to be of **product-form**
  - However, the number of active connections of different classes are **not** independent (since the normalizing constant $G$ depends on each $x_r$)
  - Only if all the links had infinite capacities,
    all the traffic classes would be independent of each other
  - Thus, it is the limited resources shared by the traffic classes
    that makes them dependent on each other

# PASTA

- Consider, for a while,
  - any simple teletraffic model (as defined in slide 15 of lecture 1) with Poisson arrivals
- According to so called **PASTA** (Poisson Arrivals See Time Averages) property,
  - arriving calls (obeying a Poisson process) see the system in the stationary state
- This is an important observation
  - applicable in many problems
- For example,
  - it allows us to calculate the end-to-end blocking probabilities in our circuit switched network model (since we assumed that new calls arrive according to a Poisson process)





# End-to-end call blocking: exact formula

- The probability that the system is (at an arbitrary time) in such a state that it cannot accept any more connections of type $r$ (that is: the end-to-end **time blocking** probability for class $r$) is clearly given by the sum

$$\sum_{\mathbf{x} \in S_r^B} \pi(\mathbf{x})$$

- But, due to the PASTA property, the end-to-end **call blocking** probability $B_r$ equals the corresponding end-to-end time blocking probability:

$$B_r = \sum_{\mathbf{x} \in S_r^B} \pi(\mathbf{x})$$

# Example

- Consider the example presented in slide 9 (and continued in slide 11)
- The end-to-end blocking probability $B_1$ for class 1 will be

$$B_1 = \pi(1,3) + \pi(2,2) + \pi(3,0) + \pi(3,1) = \frac{\frac{a_1^1 a_2^3}{1!3!} + \frac{a_1^2 a_2^2}{2!2!} + \frac{a_1^3}{3!}\left(1 + \frac{a_2^1}{1!}\right)}{\left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!} + \frac{a_2^3}{3!}\right) + \frac{a_1^1}{1!}\left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!} + \frac{a_2^3}{3!}\right) + \frac{a_1^2}{2!}\left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!}\right) + \frac{a_1^3}{3!}\left(1 + \frac{a_2^1}{1!}\right)}$$





# Approximative methods

- In practice,
  - it is extremely hard (even impossible) to apply the exact formula
  - This is due to the so called **state space explosion**: there are as many **dimensions** in the state spaces as there are routes in our model
    $\Rightarrow$ exponential growth of the state space
- Thus, **approximative** methods are needed
  - Below we will present (the simplest) one of them
- **Product Bound** method
  - estimate first blocking probabilities in each separate link (common to all traffic classes)
  - calculate then the end-to-end blocking probabilities for each class based on the hypothesis that “blocking occurs independently in each link”

# Product Bound (1)

- Consider first the blocking probability $B(j)$ in an arbitrary link $j$
  - Let $R(j)$ denote the set of routes that use link $j$
- If the capacities of all the other links (but $j$) were infinite,
  - link $j$ could be modelled as a loss system where new calls arrive according to a Poisson process with intensity $\lambda(j)$,

$$\lambda(j) = \sum_{r \in R(j)} \lambda_r$$

  - In this case, the blocking probability could be calculated from formula

$$B(j) \approx \mathrm{Erl}(n_j, \sum_{r \in R(j)} a_r)$$

  - Note that this is really an approximation, since the traffic offered to link $j$ is smaller due to blockings in other links (and not even of Poisson type).





# Product Bound (2)

- Consider then the end-to-end call blocking probability $B_r$ for class $r$
  - Let $J(r)$ denote the set of the links that belong to route $r$
  - Note that an arriving call of class $r$ will not be blocked, if it is not blocked in any link $j \in J(r)$
- If blocking occured independently in each link,
  - an arriving call of class $r$ would be blocked with probability

$$B_r \approx 1 - \prod_{j \in J(r)} (1 - B(j))$$

  - Note that for (very) small values of $B(j)$'s, we can use the following approximation:

$$B_r \approx \sum_{j \in J(r)} B(j)$$

# Contents

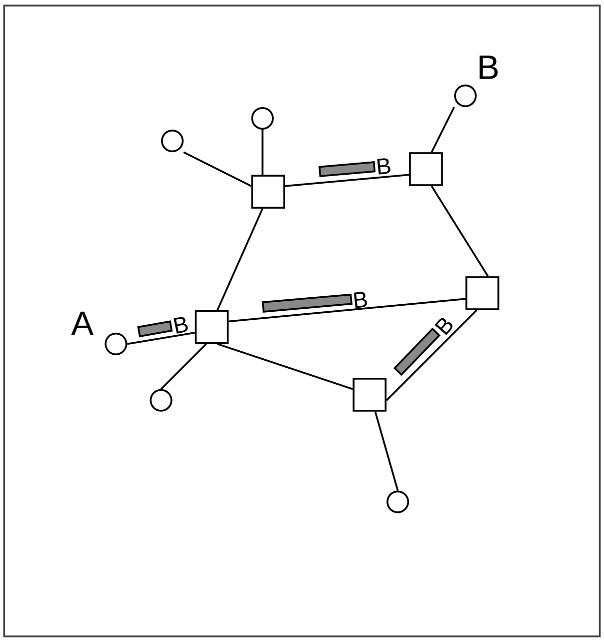
- Circuit switched network modelled as a loss network
- Packet switched network modelled as a queueing network





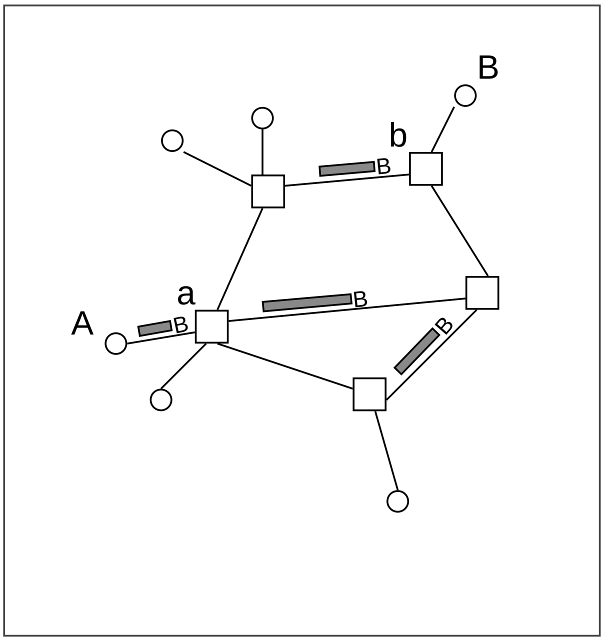
# Teletraffic model of a connectionless packet switched network (1)

- Consider a connectionless packet switched network
  - e.g. an Internet subnetwork
- Traffic:
  - data packets
  - identified by their source (A) and destination (B)
- System:
  - workstations & servers (terminals)
  - routers (network nodes)
  - access links (from terminals to routers)
  - trunks (between routers)



# Teletraffic model of a connectionless packet switched network (2)

- Quality of service:
  - described by the **average end-to-end delay** (the mean time to get from the source (A) to the destination (B))
- However, in our model
  - we restrict ourselves to the **average trunk network delay** (the mean time to get from the source router (a) to the destination router (b))
  - implicitly, we assume that the delay due to access network is negligible (or, at least, almost deterministic)





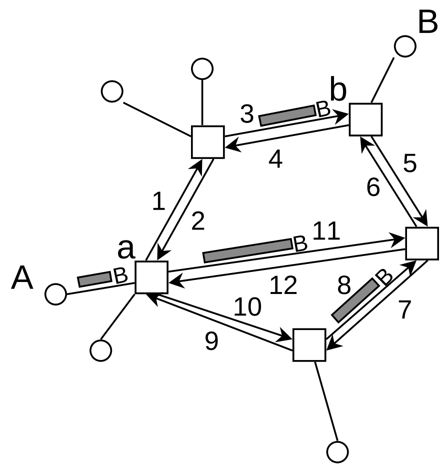



# Delay components

- Trunk network delay consists of
  - propagation delays (in links)
  - transmission delays (in links)
  - processing delays (in nodes)
  - queueing delays (before transmission and before processing)
- Note that
  - propagation and transmission delays are deterministic,
  - processing delays might be random, and
  - queueing delays are surely random
- In our model,
  - we will take into account the transmission and the related queueing delays
  - but we will ignore the propagation delays in links and the delays in nodes (the processing and the related queueing delays)

# Links j = 1,…,J

- In this case we separate the directions so that
  - all links are **one-way** (why?)
- We index the links in the trunk network by
  - $j = 1,\ldots,J$
- In the example on the right:
  - $J = 12$
- Let
  - $C_j$ = capacity of link $j$ (in bps)

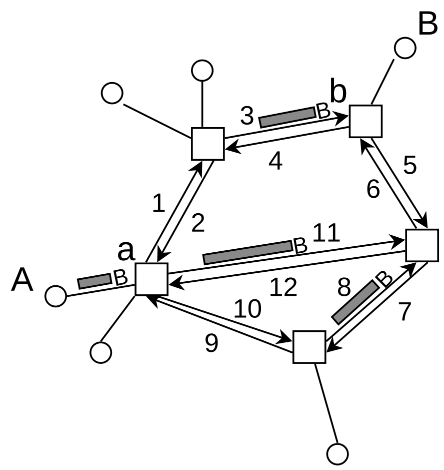






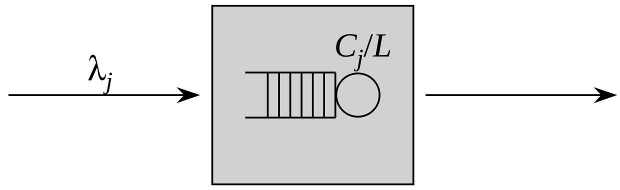

# Routes r = 1,…,R

- We define here a **route** as an
  - ordered set of (one-way) links connecting two network nodes (called origin and destination)
- We index the routes by
  - $r = 1,\ldots,R$
- In the example on the right:
  - there are three routes **from** node a **to** node b: (1,3), (11,6), (10,8,6)
  - for these routes, node a is the origin and node b is the destination
  - $R = 2*(12+10+7+3) = 64$



# Individual link model

- Each link is modelled as a
  - **pure waiting system** (with a single server and an infinite buffer)
- Let
  - $\lambda_j$ = arrival rate of packets to be transmitted on link $j$ (in packets/s)
  - $L$ = mean packet length (in bits)
  - $1/\mu_j = L/C_j$ = average packet transmission time on link $j$ (in seconds)
- Stability requirement: $\lambda_j < \mu_j$







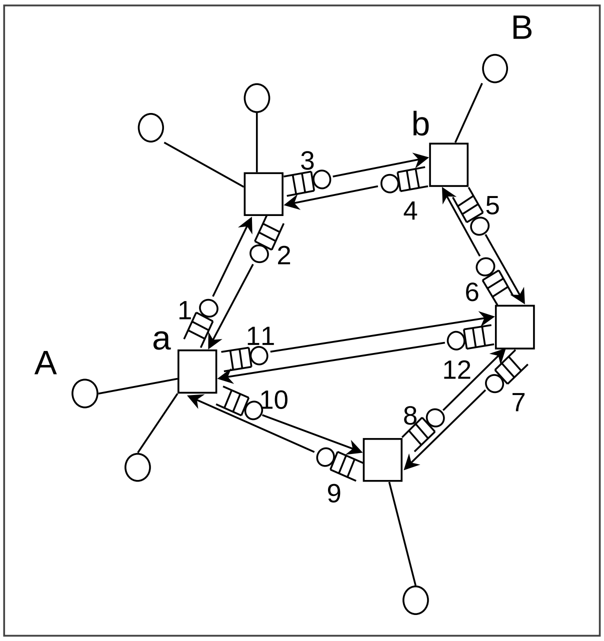

# Packet arrival rates in links

- Let
  - $\lambda(r)$ = arrival rate of packets following route $r$
  - $R(j)$ = the set of routes that use link $j$
- It follows that

$$\lambda_j = \sum_{r \in R(j)} \lambda(r)$$

# Queueing network model

- Note:
  - Average end-to-end delay is equal for all the packets following the same route
- Thus,
  - the traffic class of a packet is determined by the route *r* that the connection follows








# State space S

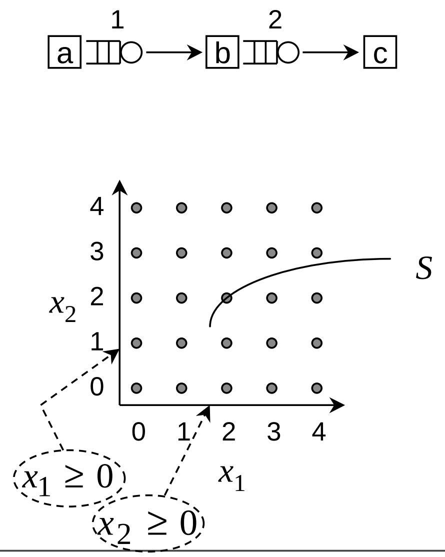
- Let
  - $x_j$ = nr of packets in queue $j$ (including the packet being transmitted (if any))
  - $\mathbf{x} = (x_1,\ldots,x_J)$
- Vector $\mathbf{x}$ is called the **state** of the system
  - A more detailed state description (including the position and traffic class of each packet in the whole system) is not needed under the assumptions that we will make later!
- In this case,
  - $x_j$ can have any non-negative value
- Thus, the **state space** $S$ is

$$S = \{\mathbf{x} \geq 0\}$$

- Note that, set $S$ is now **infinite**

# Example

- 2 links:
  - link a-b
  - link b-c
- 3 routes:
  - route a-b
  - route b-c
  - route a-b-c
- State space:
  - $S =$ {(0,0),
    (1,0),(0,1),
    (2,0),(1,1),(0,2),
    (3,0),(2,1),(1,2),(0,3),
    ...}








# Stationary link state probabilities (1)

- Assume that
  - new packets following route $r$ arrive (independently) according to a Poisson process with intensity $\lambda(r)$
  - packet lengths are independently and exponentially distributed with mean $L$
- It follows that
  - new packets to be transmitted on link $j$ arrive (independently) according to a Poisson process with intensity $\lambda_j$, where

$$\lambda_j = \sum_{r \in R(j)} \lambda(r)$$

  - packet transmission times are independently and exponentially distributed with mean $1/\mu_j = L/C_j$

## Stationary link state probabilities (2)

- Assume further that
  - the system is **stable**: $\lambda_j < \mu_j$ for all $j$
  - packet length is independently redrawn (from the same distribution) every time the packet moves from one link to another
    - This is so called **Kleinrock's independence assumption**
- Under these assumptions, it is possible to show that
  - the stationary state probability $\pi(\mathbf{x})$ for any state $\mathbf{x} \in S$ is as follows:

$$\pi(\mathbf{x}) = \prod_{j=1}^{J} (1-\rho_j)\rho_j^{x_j}$$

where $\rho_j$ denotes the traffic load of link $j$:

$$\rho_j = \frac{\lambda_j}{\mu_j} = \frac{\lambda_j L}{C_j} < 1$$



3. Modelling of telecommunication systems (part 2)

## Stationary link state probabilities (3)

- Probability $\pi(\mathbf{x})$ is again said to be of **product-form**
  - Now, the number of packets in different queues are **independent** (why?)
- Each individual queue $j$ behaves as an M/M/1 queue
  - Number of packets in queue $j$ follows a geometric distribution with mean

$$\bar{X}_j = \frac{\rho_j}{1-\rho_j}$$

## Average trunk network delay

- Consider then the average trunk network delay for class $r$
  - Let $J(r)$ denote the set of the links that belong to route $r$
- In our model, the average trunk network delay will be
  - the sum of average delays experienced in the links along the route (including **both** the transmission delay **and** the queueing delay)
- By Little's formula, the average link delay is

$$\overline{T}_j = \frac{\overline{X}_j}{\lambda_j} = \frac{1}{\lambda_j} \cdot \frac{\rho_j}{1-\rho_j} = \frac{1}{\mu_j - \lambda_j}$$

- Thus, the average trunk network delay for class $r$ is

$$\overline{T}(r) = \sum_{j \in J(r)} \overline{T}_j = \sum_{j \in J(r)} \frac{1}{\mu_j - \lambda_j}$$





## THE END