

1. a) Consider a link between two telephone exchanges with 5 parallel channels. The traffic consists of the ongoing telephone calls on the link. Each call occupies one channel. Model this as a pure loss system. Assume that new calls arrive according to a Poisson process at rate 2 calls per minute. Assume further that call holding times are independently and identically distributed with mean 3 minutes. Calculate the three different traffic intensities (traffic offered, traffic carried, and traffic lost).
 - b) Consider the processor of a packet router in a packet switched data network. The traffic consists of data packets to be processed. Model this as a pure waiting system with a single server and an infinite buffer. Assume that new packets arrive according to a Poisson process at rate 2 packets per millisecond (ms). Assume further that packet processing times are independently and exponentially distributed with mean 0.4 ms. Determine the traffic load of this system. What is the probability that an arriving packet will be processed immediately after the arrival (without any waiting delays)? What is the probability that it has to wait longer than 2 ms?
2. a) Let (as in slide 32 of lecture 1)

$$\text{Erl}(n, a) = \frac{a^n/n!}{\sum_{i=0}^n a^i/i!}$$

Derive the following recursive formula:

$$\begin{aligned} \text{Erl}(0, a) &= 1 \\ \text{Erl}(n, a) &= \frac{1}{1 + \frac{n}{a \cdot \text{Erl}(n-1, a)}} \end{aligned}$$

(*Tip*: Calculate first the quotient $\text{Erl}(n, a)/\text{Erl}(n-1, a)$.)

- b) Implement this recursive algorithm with Mathematica (or Matlab or C). Calculate the (minimum) required capacity n as a function of traffic a for $a = 1, 3, 10, 30$ and 100 erlangs given that the call blocking probability (i.e. $\text{Erl}(n, a)$) be less than 1%. Furthermore, calculate the ratio n/a in all cases.
3. Consider a subnetwork of the trunk network of some packet switched data network. There are 4 nodes that connect this subnetwork with the rest of the trunk network. Measurements tell that
 - (i) there are 1000 packets in this subnetwork (on the average) and
 - (ii) new packets (that are routed through the subnetwork) arrive to the four connecting nodes with (average) rates $\lambda_1 = 200$ packets/s, $\lambda_2 = 300$ packets/s, $\lambda_3 = 400$ packets/s and $\lambda_4 = 500$ packets/s.

What is the average time that a packet routed through the subnetwork stays there?

(*Tip*: Little's formula.)