HELSINKI UNIVERSITY OF TECHNOLOGY **Exercise 5**
Laboratory of Telecommunications Technology 27.10.1999
S-38.145 Introduction to Teletraffic Theory, Fall 1999 Aalto/Nyberg

1. Let X_1 and X_2 be two independent and identically distributed (IID) random variables, $X_i \sim \text{Exp}(\lambda)$, $\lambda > 0$ $(i = 1, 2)$. Let then

$$Z = X_1 + X_2.$$

   a) Determine the value set, probability density function (pdf), probability distribution function (PDF), mean value $E[Z]$, and variance $D^2[Z]$ of random variable Z.

   b) Assume then that $\lambda = 1$, and calculate the standard deviation $D[Z]$ and the coefficient of variation $C[Z]$ of Z.

   (*Tip:* Derive first the PDF starting from the equation $P\{Z \le z\} = \int_0^z P\{X_1 \in dx, X_2 \le z - x\}$ and utilizing the independence. The pdf is the derivative of the PDF.)

2. Consider a pure loss system with two identical servers. Service times are assumed to be independent and exponentially distributed with mean $1/\mu > 0$. Assume further that the system is empty at time 0 when two new customers arrive (together). The service of both of these customers starts immediately. After these two customers, no new customers enter the system. Let T denote the time until the system is totally empty again. What is the mean value of this random variable T?

   (*Tip:* T is the maximum of two independent random variables.)

3. Consider a pure waiting system with two identical servers. Service times are assumed to be independent and exponentially distributed with mean $1/\mu > 0$ (seconds).

   a) Assume first that there is only one customer in the system at time 0 when a new customer arrives. Thus, the service of the "new" customer starts immediately. Assume further that the "old" customer has already been served for x (seconds). What is the probability that the "new" customer leaves the system last?

   b) Assume then that there are already two customers in the system at time 0 when a new customer arrives. Thus, the "new" customer has to wait until one of the "old" customers leaves the system. Assume further that both of the "old" customer have already been served for x (seconds). What is the probability that the "new" customer leaves the system last?

   (*Tip:* Utilize the memoryless property of exponential distribution.)