Exercise 6
3.11.1999
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1. Consider the following three-state Markov processes (that is: with state space $S = \{0, 1, 2\}$). The processes are defined by giving all the state transition rates q_{ij}, $i \neq j$, in the following table ("0" means that $q_{ij} = 0$, and "+" means that $q_{ij} > 0$).

(i, j)	(0,1)	(0,2)	(1,0)	(1,2)	(2,0)	(2,1)
a)	+	0	0	+	+	+
b)	+	0	0	+	+	0
c)	+	0	0	+	0	0
d)	0	0	+	0	0	+
e)	+	0	+	+	0	+
f)	+	+	+	+	+	+

   Draw the state transition diagram for each process. Which processes are irreducible?

2. Consider still the Markov processes defined in the previous problem. We refine their definitions by giving more explicitly the state transition rates q_{ij} in the following table:

(i, j)	(0,1)	(0,2)	(1,0)	(1,2)	(2,0)	(2,1)
a)	2	0	0	2	1	1
b)	2	0	0	1	1	0
c)	1	0	0	1	0	0
d)	0	0	1	0	0	2
e)	1	0	1	1	0	2
f)	1	2	2	1	1	2

   Which processes are irreducible *and* positively recurrent (that is: Which of them have an equilibrium distribution)? Determine the equilibrium distribution (whenever it exists). Which processes are reversible (that is: Which of them satisfy all the local balance equations)?