HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 10**
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S-38.145 Introduction to Teletraffic Theory, Fall 1999 | Aalto/Nyberg

1. Consider a *symmetric* telephone network with two hierarchial levels. Assume that there are n_1 (higher level 1) areal exchanges completely connected to each other with two-way links (fully meshed topology). What is the total number l_1 of links at this level? Assume further that, for each areal exchange, there are n_2 (lower level 2) local exchanges connected to the areal exchange with two-way links (star topology). What is the total number l_2 of links at this level? All the subscribers are connected to the local exchanges, so that the areal exchanges are just used as transit exchanges. Let $T = T(i, j)$ denote the traffic matrix, where $T(i, j)$ tells the offered traffic originating from local exchange i and destined to local exchange j. Assume finally that

$$T(i,j) = \begin{cases} t_1, & \text{if local exchanges } i \text{ and } j \text{ are connected to } \textit{different} \text{ areal exchanges,} \\ t_2, & \text{otherwise.} \end{cases}$$

Note, in particular, that $T(i, i) = t_2$ for all i. What is the total offered traffic a generated from the subscribers of any single local exchange?

Note: In problems 2–4, we assume that $n_1 = 3$, $n_2 = 9$, and $t_1 = t_2 = 0.1$ erlang.

2. (Node dimensioning) Assume that the mean holding time is $h = 3$ minutes. What is the rate λ_i of call requests arriving at a level-i node, $i = 1, 2$? Dimension the nodes so that the traffic load $\rho < 0.5$ in all nodes.

3. (Link dimensioning) What is the traffic a_i offered to a level-i link, $i = 1, 2$? Dimension the links so that the call blocking probability $B \leq 1\%$ in all links.

4. (Network optimization) According to so called *Product Bound* method, the end-to-end blocking probability B_e for a route consisting of links $j = 1, \ldots, J$, can be approximately calculated from the following equation:

$$B_e = 1 - (1 - B(1)) \cdots (1 - B(J)),$$

where $B(j)$ refer to the call blocking probability for link j, $j = 1, \ldots, J$. When all $B(j)$'s are small we have

$$B_e \approx B(1) + \ldots + B(J).$$

Using this equation, we can deduce that the dimensioning made in the previous problem guarantees that the end-to-end blocking probability between any two local exchanges is at most 3%.

However, this is not the only (and neither the best) possibility to allocate resources so that the same target Grade of Service is guaranteed. A better allocation can be determined by utilizing so called *Moe's principle*. Dimension first all the links so that the call blocking probability $B < 3\%$ in all links. After that increase the link capacities step-by-step. In each step, add one more channel to *all* the links at level i, for which the figure H_i,

$$H_i = \frac{\mathrm{Erl}(C_i, a_i) - \mathrm{Erl}(C_i + 1, a_i)}{l_i}$$

is greatest. Here C_i refers to the current number of channels allocated to a link at level i, $i = 1, 2$. Stop the allocation as soon as the end-to-end blocking probability between any two local exchanges becomes smaller than (or equal to) 3%. Compare the total amount of allocated channels in problems 3 and 4.