



3. Modelling of telecommunication systems (part 2)

lect03.ppt

S-38.145 - Introduction to Teletraffic Theory - Fall 1999

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3. Modelling of telecommunication systems (part 2)

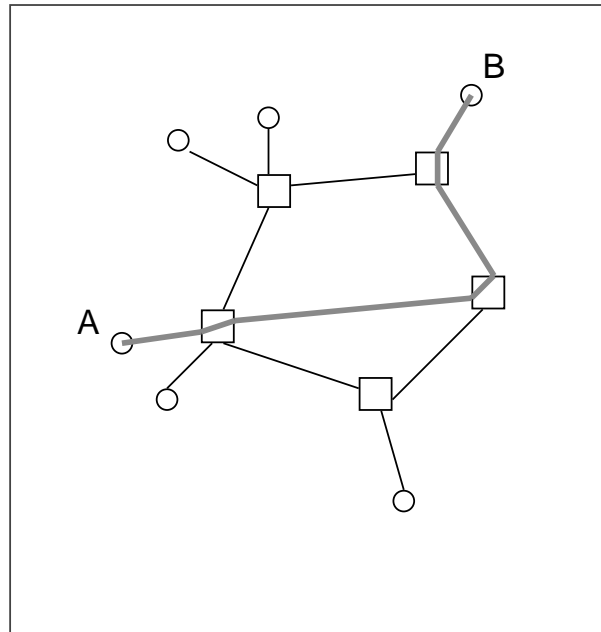
Contents

- Circuit switched network modelled as a loss network
- Packet switched network modelled as a queueing network

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Teletraffic model of a circuit switched network (1)

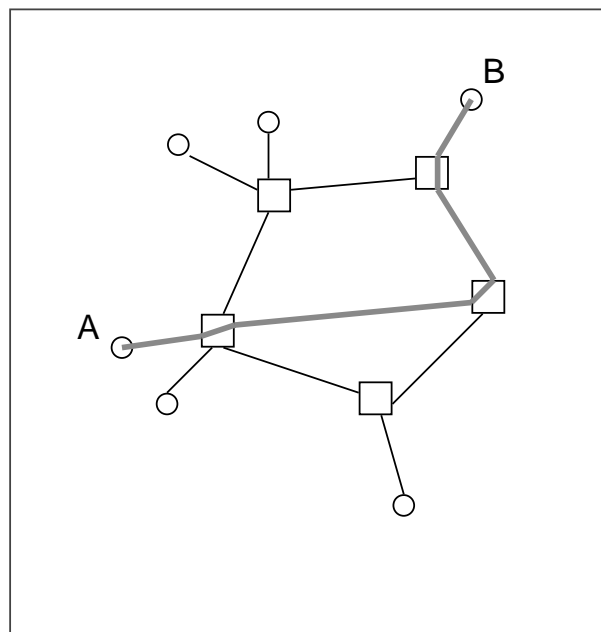
- Consider a circuit switched network
 - e.g. a telephone network
- Traffic:
 - telephone calls
 - each (carried) call occupies one channel on each link among its route
- System:
 - telephone machines (terminals)
 - exchanges (network nodes)
 - access links (from terminals to exchanges)
 - trunks (between exchanges)



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Teletraffic model of a circuit switched network (2)

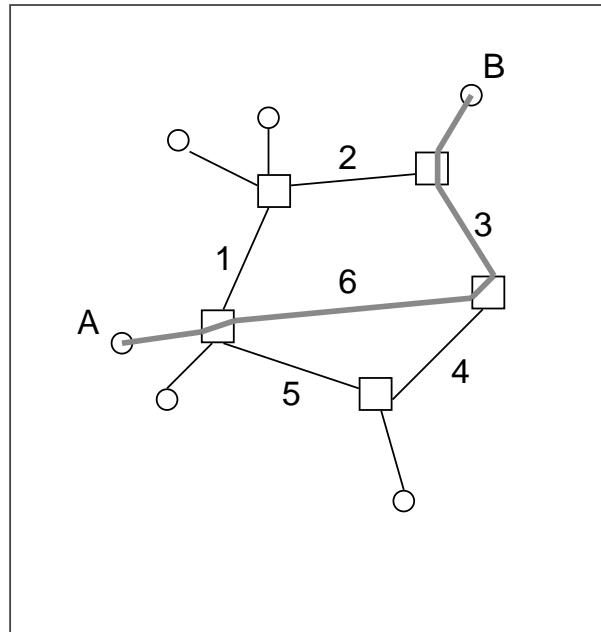
- Quality of service:
 - described by the **end-to-end call blocking probability** (prob. that a desired connection cannot be set up due to congestion along the route of the connection)
- In our model we assume that
 - the network nodes and the whole access network are non-blocking
- Thus, a call is blocked
 - if and only if all channels are occupied in any trunk network link along the route of that call



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Links $j = 1, \dots, J$

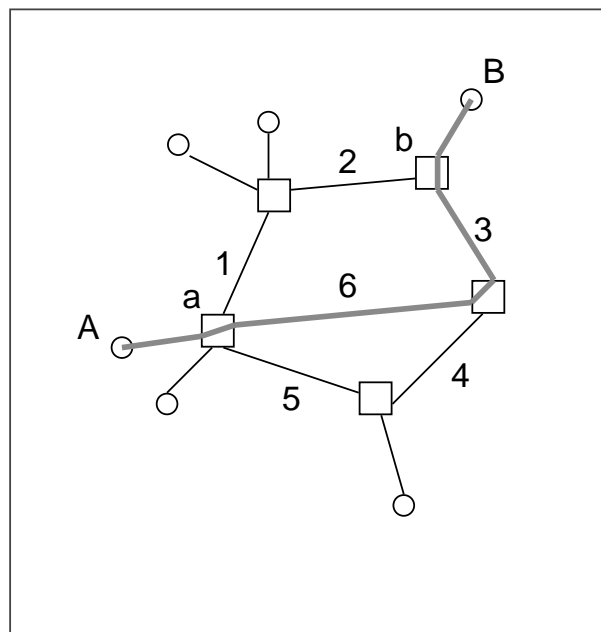
- In our model,
 - all links are **two-way** (why?)
- We index the links in the trunk network by
 - $j = 1, \dots, J$
- In the example on the right:
 - $J = 6$
- Let
 - $n_j =$ nr of channels in link j (that is: the link capacity)
 - $\mathbf{n} = (n_1, \dots, n_J)$
- Each link is modelled as a
 - **pure loss system**



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Routes $r = 1, \dots, R$

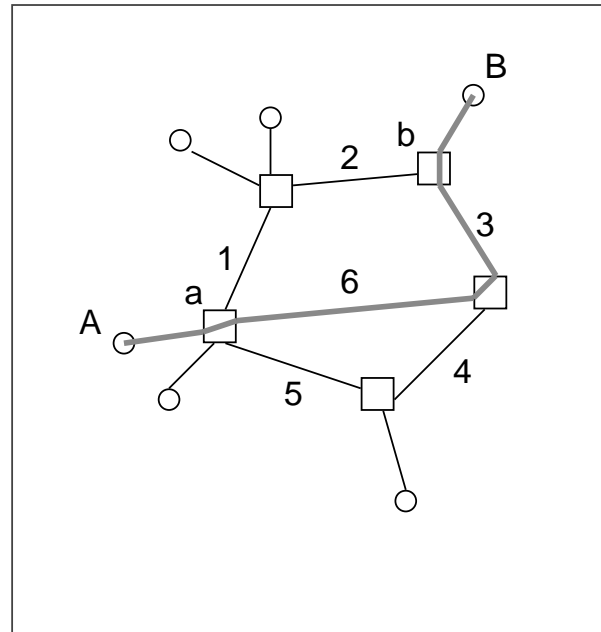
- We define a **route** as a
 - set of (two-way) links connecting two network nodes
- We index the routes by
 - $r = 1, \dots, R$
- In the example on the right:
 - there are three routes **between** nodes a and b: $\{1,2\}, \{6,3\}, \{5,4,3\}$
 - $R = 12 + 10 + 7 + 3 = 32$
- Let
 - $d_{jr} = 1$ if link j belongs to route r
 - $d_{jr} = 0$ otherwise
 - $\mathbf{D} = (d_{jr} \mid j = 1, \dots, J; r = 1, \dots, R)$



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Loss network model

- Note:
 - End-to-end call blocking prob. is equal for all the connections following the same route
- Thus,
 - the traffic class of a connection is determined by the route r that the connection follows
- Let
 - x_r = number of active connections following route r
 - $\mathbf{x} = (x_1, \dots, x_R)$
- Vector \mathbf{x} is called
 - the **state** of the system



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State space S

- The number of active connections x_r for any traffic class r is limited by the link capacities n_j along the corresponding route r :

$$\sum_{r=1}^R d_{jr} x_r \leq n_j \quad \text{for all } j$$

- The same in vector form:

$$\mathbf{D} \cdot \mathbf{x} \leq \mathbf{n}$$

- Thus, the **state space** S (that is: the set of **admissible** states) is

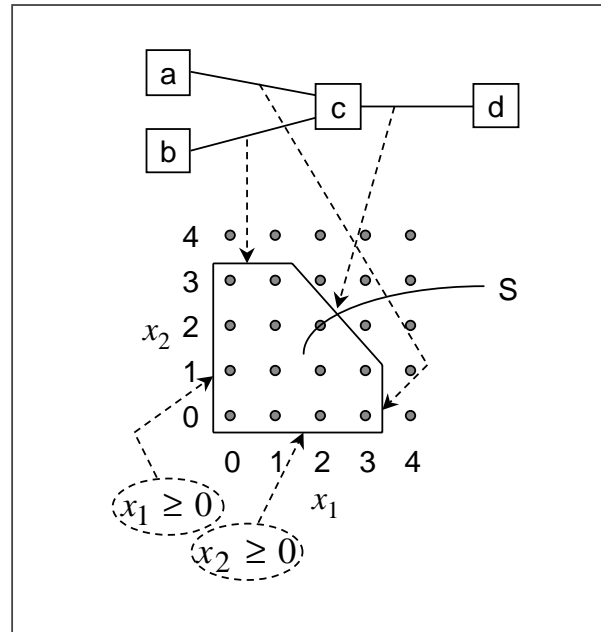
$$S = \{\mathbf{x} \geq 0 \mid \mathbf{D} \cdot \mathbf{x} \leq \mathbf{n}\}$$

- Note that, due to finite link capacities, set S is **finite**

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Example

- 3 links with capacities:
 - link a-c: 3 channels
 - link b-c: 3 channels
 - link c-d: 4 channels
- 2 routes:
 - route a-c-d
 - route b-c-d
 - The other 4 routes (which?) are ignored in this model
- State space:
 - $S = \{$
 - $(0,0), (0,1), (0,2), (0,3),$
 - $(1,0), (1,1), (1,2), (1,3),$
 - $(2,0), (2,1), (2,2),$
 - $(3,0), (3,1)\}$



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Set S_r of non-blocking states for class r

- Consider
 - an arriving call belonging to class r (that is: following route r)
- It will **not** be blocked by link j belonging to route r
 - if there is at least one free channel on link j :

$$\sum_{r'=1}^R d_{jr'} x_{r'} \leq n_j - 1 \quad \text{for all } j \in r$$

- The same in vector form (\mathbf{e}_r being here the unit vector in direction r):

$$\mathbf{D} \cdot (\mathbf{x} + \mathbf{e}_r) \leq \mathbf{n}$$

- The set S_r of **non-blocking** states is thus

$$S_r = \{\mathbf{x} \geq 0 \mid \mathbf{D} \cdot (\mathbf{x} + \mathbf{e}_r) \leq \mathbf{n}\}$$

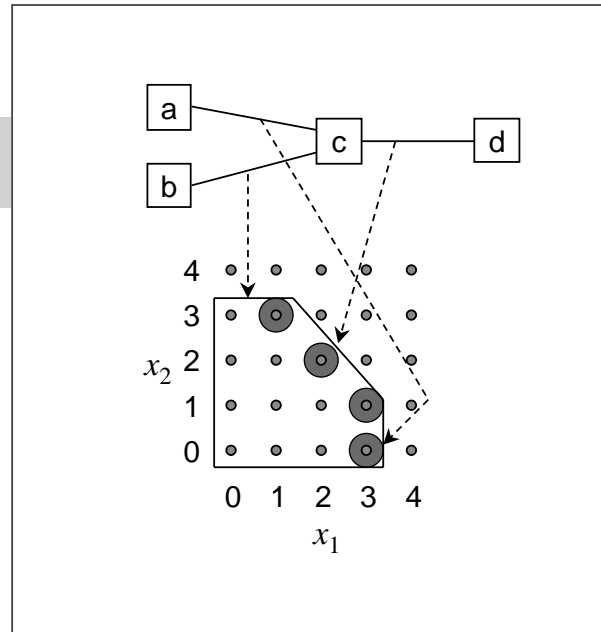
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Set S_r^B of blocking states for class r

- The set S_r^B of **blocking** states for class r is clearly:

$$S_r^B = S \setminus S_r$$

- Summary:
 - an arriving call of class r is blocked (and lost) if and only if the state x of the system belongs to set S_r^B
- Example (continued):
 - The blocking states S_1^B for connections of class 1 (using route a-c-d) are circled in the figure
 - $S_1^B = \{ (1,3), (2,2), (3,0), (3,1) \}$



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Stationary state probabilities (1)

- Assume that
 - new connection requests belonging to traffic class r arrive (independently) according to a Poisson process with intensity λ_r
 - call holding times independently and identically distributed with mean h
- Denote
 - $a_r = \lambda_r h$ (traffic intensity for class r)

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Stationary state probabilities (2)

- Then it is possible to show that
 - the stationary state probability $\pi(\mathbf{x})$ for any state $\mathbf{x} \in S$ is as follows:

$$\pi(\mathbf{x}) = G^{-1} \cdot \prod_{r=1}^R f_r(x_r)$$

where G is a normalizing constant:

$$G = \sum_{\mathbf{x} \in S} \prod_{r=1}^R f_r(x_r)$$

and the functions $f_r(x_r)$ are defined as follows:

$$f_r(x_r) = \frac{a_r^{x_r}}{x_r!}$$

Stationary state probabilities (3)

- Probability $\pi(\mathbf{x})$ is said to be of **product-form**
 - However, the number of active connections of different classes are **not** independent (since the normalizing constant G depends on each x_r)
 - Only if all the links had infinite capacities, all the traffic classes were independent of each other
 - Thus, it is the limited resources shared by the traffic classes that makes them dependent on each other

PASTA

- Consider, for a while,
 - any simple teletraffic model (as defined in slide 15 of lecture 1) with Poisson arrivals
- According to so called **PASTA** (Poisson Arrivals See Time Averages) property,
 - arriving calls (obeying a Poisson process) see the system in the stationary state
- This is an important observation
 - applicable in many problems
- For example,
 - it allows us to calculate the end-to-end blocking probabilities in our circuit switched network model (since we assumed that new calls arrive according to a Poisson process)

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End-to-end call blocking: exact formula

- The probability that the system is (at an arbitrary time) in such a state that it cannot accept any more connections of type r (that is: the end-to-end time blocking probability for class r) is clearly given by the sum

$$\sum_{\mathbf{x} \in S_r^B} \pi(\mathbf{x})$$

- But, due to the PASTA property, the end-to-end call blocking probability B_r is equal to the corresponding end-to-end time blocking probability:

$$B_r = \sum_{\mathbf{x} \in S_r^B} \pi(\mathbf{x})$$

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Example

- Consider the example presented in slide 9 (and continued in slide 11)
- The end-to-end blocking probability B_1 for class 1 will be

$$B_1 = \pi(1,3) + \pi(2,2) + \pi(3,0) + \pi(3,1) =$$

$$\frac{a_1^1 a_2^3}{1!3!} + \frac{a_1^2 a_2^2}{2!2!} + \frac{a_1^3}{3!} \left(1 + \frac{a_2^1}{1!} \right)$$

$$\left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!} + \frac{a_2^3}{3!} \right) + \frac{a_1^1}{1!} \left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!} + \frac{a_2^3}{3!} \right) + \frac{a_1^2}{2!} \left(1 + \frac{a_2^1}{1!} + \frac{a_2^2}{2!} \right) + \frac{a_1^3}{3!} \left(1 + \frac{a_2^1}{1!} \right)$$

Approximative methods

- In practice,
 - it is extremely hard (even impossible) to apply the exact formula
 - This is due to the so called **state space explosion**: there are as many **dimensions** in the state spaces as there are routes in our model
 ⇒ exponential growth of the state space
- Thus, **approximative** methods are needed
 - Below we will present (the simplest) one of them
- **Product Bound** method
 - estimate first blocking probabilities in each separate link (common to all traffic classes)
 - calculate then the end-to-end blocking probabilities for each class based on the hypothesis that “blocking occurs independently in each link”

Product Bound (1)

- Consider first the blocking probability $B(j)$ in an arbitrary link j
 - Let $R(j)$ denote the set of routes that use link j
- If the capacities of all the other links (but j) were infinite,
 - link j could be modelled as a loss system where new calls arrive according to a Poisson process with intensity $\lambda(j)$,

$$\lambda(j) = \sum_{r \in R(j)} \lambda_r$$

- In this case, the blocking probability could be calculated from formula

$$B(j) \approx \text{Erl}(n_j, \sum_{r \in R(j)} a_r)$$

- Note that this is really an approximation, since the traffic offered to link j is smaller due to blockings in other links (and not even of Poisson type).

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Product Bound (2)

- Consider then the end-to-end call blocking probability B_r for class r
 - Let $J(r)$ denote the set of the links that belong to route r
 - Note that an arriving call of class r will not be blocked, if it is not blocked in any link $j \in J(r)$
- If blocking would occur independently in each link,
 - an arriving call of class r would be blocked with probability

$$B_r \approx 1 - \prod_{j \in J(r)} (1 - B(j))$$

- Note that for (very) small values of $B(j)$'s, we can use the following approximation:

$$B_r \approx \sum_{j \in J(r)} B(j)$$

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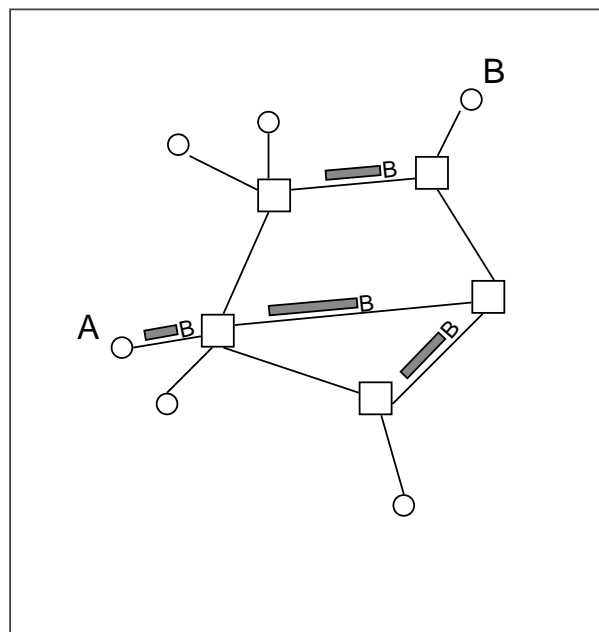
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Teletraffic model of a connectionless packet switched network (1)

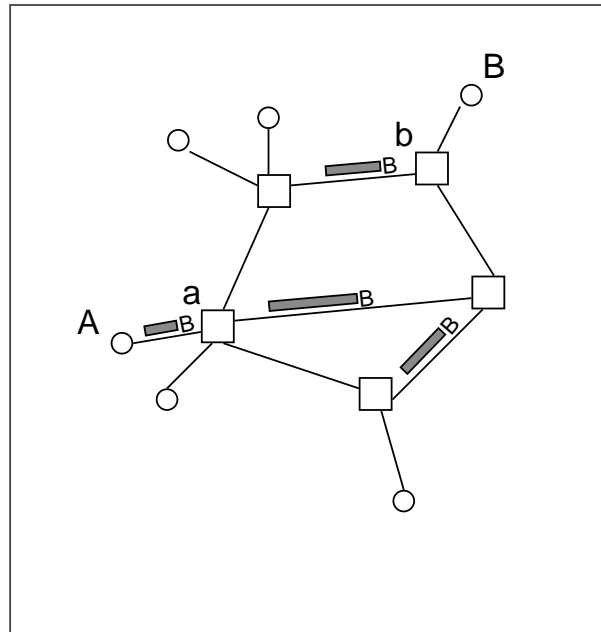
- Consider a connectionless packet switched network
 - e.g. an Internet subnetwork
- Traffic:
 - data packets
 - identified by their source (A) and destination (B)
- System:
 - workstations & servers (terminals)
 - routers (network nodes)
 - access links (from terminals to routers)
 - trunks (between routers)



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Teletraffic model of a connectionless packet switched network (2)

- Quality of service:
 - described by the **average end-to-end delay** (the mean time to get from the source (A) to the destination (B))
- However, in our model
 - we restrict ourselves to the **average trunk network delay** (the mean time to get from the source router (a) to the destination router (b))
 - implicitly, we assume that the delay due to access network is negligible (or, at least, almost deterministic)



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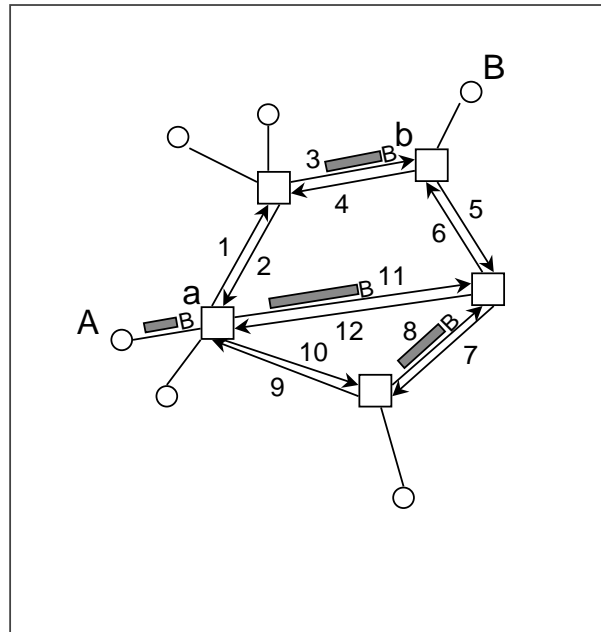
Delay components

- Trunk network delay consists of
 - propagation delays (in links)
 - transmission delays (in links)
 - processing delays (in nodes)
 - queueing delays (before transmission and before processing)
- Note that
 - propagation and transmission delays are deterministic,
 - processing delays might be random, and
 - queueing delays are surely random
- In our model,
 - we will take into account the transmission and the related queueing delays
 - but we will ignore the propagation delays in links and the delays in nodes (the processing and the related queueing delays)

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Links $j = 1, \dots, J$

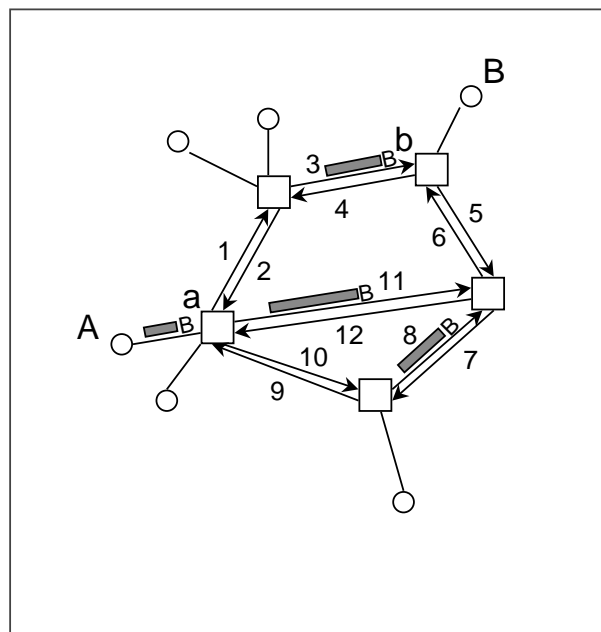
- In this case we separate the directions so that
 - all links are **one-way** (why?)
- We index the links in the trunk network by
 - $j = 1, \dots, J$
- In the example on the right:
 - $J = 12$
- Let
 - $C_j = \text{capacity of link } j \text{ (in bps)}$



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Routes $r = 1, \dots, R$

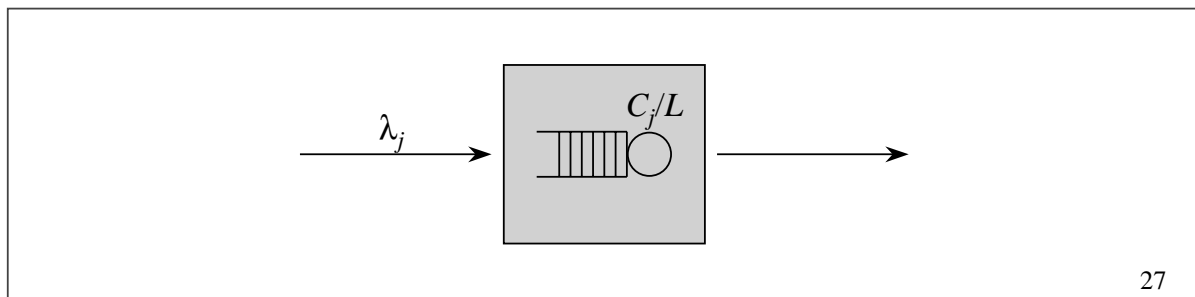
- We define here a **route** as an
 - ordered set of (one-way) links connecting two network nodes (called origin and destination)
- We index the routes by
 - $r = 1, \dots, R$
- In the example on the right:
 - there are three routes **from** node a **to** node b: (1,3), (11,6), (10,8,6)
 - for these routes, node a is the origin and node b is the destination
 - $R = 2 * (12 + 10 + 7 + 3) = 64$



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Individual link model

- Each link is modelled as a
 - **pure waiting system** (with a single server and an infinite buffer)
- Let
 - λ_j = arrival rate of packets to be transmitted on link j (in packets/s)
 - L = mean packet length (in bits)
 - $1/\mu_j = L/C_j$ = average packet transmission time on link j (in seconds)
- Stability requirement: $\lambda_j < \mu_j$



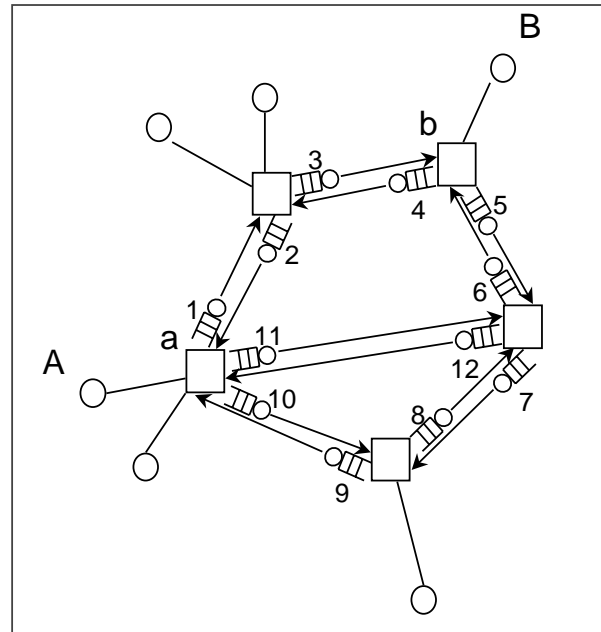
Packet arrival rates in links

- Let
 - $\lambda(r)$ = arrival rate of packets following route r
 - $R(j)$ = the set of routes that use link j
- It follows that

$$\lambda_j = \sum_{r \in R(j)} \lambda(r)$$

Queueing network model

- Note:
 - Average end-to-end delay is equal for all the packets following the same route
- Thus,
 - the traffic class of a packet is determined by the route r that the connection follows



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State space S

- Let
 - x_j = nr of packets in queue j (including the packet being transmitted (if any))
 - $\mathbf{x} = (x_1, \dots, x_J)$
- Vector \mathbf{x} is called the **state** of the system
 - A more detailed state description (including the position and traffic class of each packet in the whole system) is not needed under the assumptions that we will make later!
- In this case,
 - x_j can have any non-negative value
- Thus, the **state space** S is

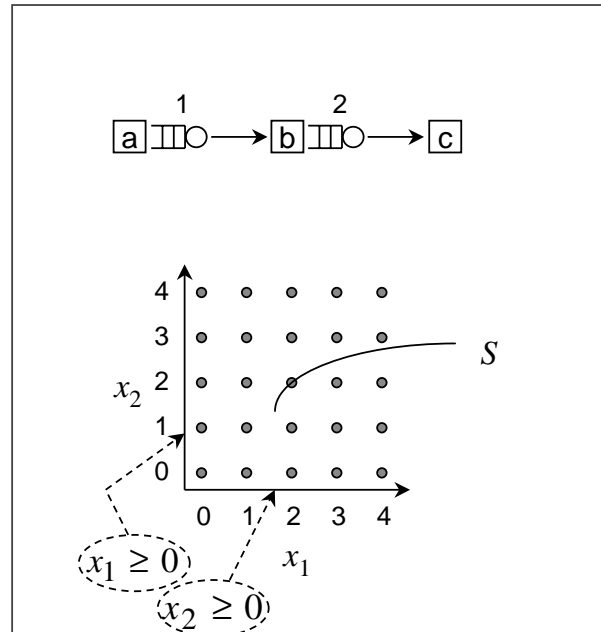
$$S = \{\mathbf{x} \geq 0\}$$

- Note that, set S is now **infinite**

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Example

- 2 links:
 - link a-b
 - link b-c
- 3 routes:
 - route a-b
 - route b-c
 - route a-b-c
- State space:
 - $S = \{$
 - $(0,0),$
 - $(1,0), (0,1),$
 - $(2,0), (1,1), (0,2),$
 - $(3,0), (2,1), (1,2), (0,3),$
 - $\dots\}$



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Stationary link state probabilities (1)

- Assume that
 - new packets following route r arrive (independently) according to a Poisson process with intensity $\lambda(r)$
 - packet lengths are independently and exponentially distributed with mean L
- It follows that
 - new packets to be transmitted on link j arrive (independently) according to a Poisson process with intensity λ_j , where

$$\lambda_j = \sum_{r \in R(j)} \lambda(r)$$

- packet transmission times are independently and exponentially distributed with mean $1/\mu_j = L/C_j$

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Stationary link state probabilities (2)

- Assume further that
 - the system is **stable**: $\lambda_j < \mu_j$ for all j
 - packet length is independently redrawn (from the same distribution) every time the packet moves from one link to another
 - This is so called **Kleinrock's independence assumption**
- Under these assumptions, it is possible to show that
 - the stationary state probability $\pi(\mathbf{x})$ for any state $\mathbf{x} \in S$ is as follows:

$$\pi(\mathbf{x}) = \prod_{j=1}^J (1 - \rho_j) \rho_j^{x_j}$$

where ρ_j denotes the traffic load of link j :

$$\rho_j = \frac{\lambda_j}{\mu_j} = \frac{\lambda_j L}{C_j} < 1$$

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Stationary link state probabilities (3)

- Probability $\pi(x)$ is again said to be of **product-form**
 - Now, the number of packets in different queues are **independent** (why?)
- Each individual queue j behaves as an M/M/1 queue
 - Number of packets in queue j follows a geometric distribution with mean

$$\bar{X}_j = \frac{\rho_j}{1 - \rho_j}$$

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Average trunk network delay

- Consider then the average trunk network delay for class r
 - Let $J(r)$ denote the set of the links that belong to route r
- In our model, the average trunk network delay will be
 - the sum of average delays experienced in the links along the route (including **both** the transmission delay **and** the queueing delay)
- By Little's formula, the average link delay is

$$\bar{T}_j = \frac{\bar{X}_j}{\lambda_j} = \frac{1}{\lambda_j} \cdot \frac{\rho_j}{1 - \rho_j} = \frac{1}{\mu_j - \lambda_j}$$

- Thus, the average trunk network delay for class r is

$$\bar{T}(r) = \sum_{j \in J(r)} \bar{T}_j = \sum_{j \in J(r)} \frac{1}{\mu_j - \lambda_j}$$

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THE END



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