S-38.149 Home exercises of lecture 1

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- Exercise 1 (Ex. 6.1 from the book)

Consider a steady state open network with three exponential nodes (with parameter μ_1, μ_2, μ_3) and Poisson arrivals (with rate γ) to node number 1. Customers follow one of two routes through the network: node 1 to node 2 (with probability p) an node 1 to node 3 (with probability $q = 1 - p$). Write down the arrival rates λ_i at node i ($i = 1, 2, 3$). Use Little's result and Jackson's theorem to obtain the mean waiting time spent by a customer in the network and show that if $\mu_2 = \mu_3$, this is least when $p = q = \frac{1}{2}$.

- Exercise 2

Consider a Jackson network presented in the figure below. Each server is a M/M/1-system with service rate $\mu = 30$ customer/second. External arrival rate to station 1 is $\gamma = 10$ customer/second. The routing probabilities are show in the figure. Find out the probability that there is at most one customer in the system.

- Exercise 3 (Ex. 6.3 from the book)

Let $G(K)$ be the normalizing constant for a Gordon-Newell network of M servers with constant service rates $\mu_1, ..., \mu_M$, visition rates $e_1, ..., e_M$ and population K. Prove that in the steady state:

$$(a) \quad P(N_i = k | N_j \geq h) = \left(\frac{e_i}{\mu_i}\right)^k \frac{G_i(K-k-h)}{G(K-h)}$$

$$(b) \quad P(N_i = k | N_j = h) = \left(\frac{e_i}{\mu_i}\right)^k \frac{G_{ij}(K-k-h)}{G_j(K-h)},$$

where $(1 \leq i \neq j \leq M)$, N_i is the random variable for the queue size at server i and G_i, G_{ij} are the normalizing constants for the network with servers i, i and j respectively removed.