S-38.149 Homework exercises of lecture 9

Henri Koskinen

Exercise 1

Consider a balanced allocation of the physical network model, characterized by $\Phi(x)$. Show that the service capacities $\psi_1, \dots \psi_N$ of the corresponding processor-sharing queueing network model are balanced by

$$\Psi(y) = \prod_{i \in S_0} \frac{1}{y_i!} \cdot \Phi(x) \prod_{k=1}^{K} \binom{x_k}{y_i, i \in S_k}$$

Exercise 2

The following excerpt from [BP03a] contains an inconsistency. Find this inconsistency (1 point) and present a correction (1 point).

"

4.1 Fair allocations

As mentioned in Section 1, most allocations considered so far in the literature are based on the notion of *utility*. Assume the utility of a flow is an increasing and strictly concave function U of its rate. A unique allocation is then defined by maximizing the overall utility:

$$\sum_{k=1}^{K} x_k U\left(\frac{\phi_k(x)}{x_k}\right),$$

under the capacity constraints (1). We say that these allocations are "fair" in the sense that the utility function U is the same for all classes of flow. In particular, $\langle \dots \rangle$

The allocation associated with the log utility function is known as proportional fairness [14]. Another example is the range of allocations associated with the power functions $U = (\cdot)^\alpha$, where the parameter $\alpha, \alpha < 1, \alpha \neq 0$, captures the trade-off between efficiency (in terms of overall allocated capacity $\sum_{k=1}^{K} \phi_k(x)$) and fairness. Specifically, the allocation maximizes the overall capacity when $\alpha \to 1$ and tends to max-min fairness when $\alpha \to -\infty$ [23]. For convenience, we also refer to max-min fairness as a utility-based allocation.
"

[14] F.P. Kelly, *Reversibility and Stochastic Networks*, Wiley, 1979.

[23] J. Mo and J. Walrand, *Fair end-to-end window-based congestion control*, IEEE/ACM Transactions on Networking 8-5 (2000) 556-567.