TEKNILLINEN KORKEAKOULU
Teletekniikan laboratorio
S-38.149 Licenciate Course in Teletraffic Theory

Exercise 1
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1. **Decaying variance**

   Let $X_1, X_2, \ldots$ be i.i.d.[1] random variables with finite mean and variance. Define aggregated process as

   $$X_k^{(m)} = \frac{1}{m}(X_{km-m+1} + \ldots + X_{km}).$$

   Prove that for $X_K^{(m)}$ it holds

   $$\mathrm{V}\left[X_K^{(m)}\right] \sim \frac{a}{m},$$

   where a is some constant. (Process is said to have slowly decaying variance if $\mathrm{V}\left[X_K^{(m)}\right] \sim a\,m^{-\beta}, 0 < \beta < 1$ as $m \to \infty$)

2. **Heavy-tailed distributions**

   Let random variable X follow Pareto-dsitribution that is

   $$\mathrm{P}\{X > x\} = \left(\frac{k}{x}\right)^{\alpha}, \qquad x \geq k > 0 \text{ and } 0 < \alpha.$$

   Prove that

   (a) mean of X is infinite if $\alpha \leq 1$

   (b) variance of X is also infinite if $\alpha < 2$

[1] independent identically distributed