Traffic Models for Mobile Networks

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1 Introduction

The main features of the mobile networks, that make traffic models developed for the fixed networks unapplicable, are user mobility and the radio propagation environment. The former require to model a spatial distribution of the users and their movements. In the case of the cellular network, which is the only case considered here, it requires to model the handover mechanisms, that can be network specific. The propagation environment introduces a phenomena of outage due to path loss or propagation decay, and the outage probability is becoming no less important performance characteristic than the blocking probability. Because of this environment we can not entirely separate the users channels, and co-channel interference must be also taken into account, especially in the case of CDMA (Code Division Multiple Access) network. There are also network type specific techniques which require special traffic models, e.g. dynamic channel assignment in TDMA (Time Division Multiple Access) networks.

This report just gives an overview of some problems and traffic models trying to provide analytical solutions and is far from to be exhaustive. The problems and models chosen are representative samples from this wide area. However, the report is focusing on CDMA models, that are given in more details. The main task of CDMA models is to estimate the network capacity. The TDMA models address analysis of efficiency and capacity enhancements of such techniques as handover and dynamic channel assignments. To make notations, used in different papers, consistent some original symbols were changed.

2 TDMA traffic models

TDMA separates the users by time slots and frequency channels. Ignoring adjacent and co-channel interference TDMA can be considered as a typical circuit-switched loss network. Therefore all the methods developed for the loss networks apply also to TDMA. Problems requiring a special treatment in TDMA traffic modelling are handover and channel assignment.

2.1 A traffic model for handover

TDMA network control can be divided in three parts [M91]: channel assignment determines which channels are in use in each cell, call acceptance determines to which cell a new call is accepted, and handover decides in which cell the call is continued. A cellular network can be modellled as an open queuing network, where customers can migrate between the queues (cells). Then channel assignment corresponds to capacity constraints on queues and handover to restrictions on movements between cells. In [M91] a model for such a network was developed by using a fixed point approximation. This approximation is based on an assumption of independence between call arrivals to different cells, between cell blocking probabilities. Handover arrivals are modelled as Poisson processes. If there is no handover priority a following traffic equations valid:

$$\rho_j \lambda_j = \nu_j + \sum_k \rho_k \lambda_{kj} (1 - B_k) \tag{1}$$

where λ_j is a total rate of departure from cell j, ρ_j is the average number of users in cell, ν_j is the call arrival rate, and λ_{kj} is the handover rate from cell k to cell j, and B_k is a blocking probability for cell k calculated from the Erlang loss function for cell capacity C_k and traffic intensity $\rho_{k.}$ These equations have unique solutions for $B_k < 1$. They can be written as a linear system.

$$(\Lambda_r - \mathbf{R}^T \Lambda_b)\boldsymbol{\rho} = \boldsymbol{\nu} \tag{2}$$

where $\Lambda_r = diag(\lambda_j)$, $\Lambda_b = diag(1 - B_j)$, $\mathbf{R} = (\lambda_{ij})$, $\boldsymbol{\rho} = (\rho_j)$, $\boldsymbol{\nu} = (\nu_j)$. Defining $\mathbf{A} = \Lambda_r - \mathbf{R}^T \Lambda_b$ and $\mathbf{X} = \mathbf{R}^T \Lambda_b \Lambda_r^{-1}$ we get:

$$\boldsymbol{\rho} = \mathbf{A}^{-1}\boldsymbol{\nu} = \Lambda_r^{-1}(I – \mathbf{X})^{-1}\boldsymbol{\nu} = \Lambda_r^{-1}(I + \sum_{i=1}^{\infty} \mathbf{X}^i)\nu_i \tag{3}$$

If the infinite sum is truncated to just the identity matrix then we get equation for the fresh traffic without handovers $\rho_j\lambda_j = \nu_j$, if the sum is truncated to $I+\mathbf{X}+\mathbf{X}^2$ then we get an equation accounting for two handovers:

$$\rho_j\lambda_j = \nu_j + \sum_k \nu_k \frac{\lambda_{kj}}{\lambda_k}(1-B_k) + \sum_k \sum_i \nu_i \frac{\lambda_{ik}}{\lambda_i}\frac{\lambda_{kj}}{\lambda_k}(1-B_i)(1-B_k) \tag{4}$$

With a handover channel reservation policy a t_j number of channels is reserved in cell j for handover attempts only and the call is accepted if there are less than $C_j - t_j$ channels occupied. Then under assumption of exponential distribution of channel holding times the fresh call and handover blocking probabilities B_j^f and B_j^h can be determined by following formulas:

$$B_j^f = G^{-1}(\rho_f, \rho_h, c, t)\frac{(\rho_f + \rho_h)^{c-t}\rho_h^t}{c!} \tag{5}$$

$$B_j^h = G^{-1}(\rho_f, \rho_h, c, t)(\rho_f + \rho_h)^{c-t} \sum_{n=c-t}^{c} \frac{\rho_h^{n-c+t}}{n!} \tag{6}$$

where

$$G(\rho_f, \rho_h, c, t) = \sum_{n=0}^{c-t-1} \frac{(\rho_f+\rho_h)^n}{n!} + (\rho_f + \rho_h)^{c-t} \sum_{n=c-t}^{c} \frac{\rho_h^{n-c+t}}{n!} \tag{7}$$

With independent blocking assumption in equilibrium the steady-state rate of calls in and out of cell is:

$$\alpha_j = (1-B_j^f)\nu_j + (1-B_j^h)\sum \alpha_k \frac{\lambda_{kj}}{\lambda_k} \tag{8}$$

For this case the carried traffic equations can be solved as a linear system:

$$\boldsymbol{\gamma} = \mathbf{A}^{-1}\Lambda_{bf}\boldsymbol{\nu} \tag{9}$$

where $\boldsymbol{\gamma} = (\gamma_j)$ is the carried traffic matrix, $\Lambda_{bf} = diag(1-B_j^f)$, $\mathbf{A} = \Lambda_r - \mathbf{R}^T\Lambda_{bh}$ and $\Lambda_{bh} = diag(1-B_j^h)$.

The blocking probabilities can be found by iterative solution of this linear system. The call dropout probabilities are

$$\mathbf{B}^d = \sum_{k=1}^{\infty} \mathbf{p}^k = (I + \sum_{k=1}^{\infty} \mathbf{X}_k)\mathbf{p}^1 = (I-\mathbf{X})^{-1}\mathbf{p}^1 \tag{10}$$

where $\mathbf{B}^d = (B_j^d)$ and $p = (p_j^k)$, probability of failure for a call starting in cell j on the k-th handover. Hence, the probability of a call starting in cell j to terminate successfully is:

$$P_s^j = (1-B_j^d)(1-B_j^d) \tag{11}$$

2.2 Product form solution for channel assignment policies

The performance of dynamic channel assignment is difficult to evaluate analytically as it depends on the actual algorithms. However, it was proposed an ideal policy, called "maximal packing" MP, which can be treated analytically though it is not always realisable. With this policy, a call will be blocked only if there is no possible reallocation of channels, including those already reserved to other calls. Performance of this policy and its comparison with fixed channel assignment was analysed in [Ev91] by using a product form solution. This solution represents the probability of having a network state $\mathbf{n}$ for m cells in the following general form:

$$p(\mathbf{n}) = \begin{cases} p(\mathbf{0})\frac{A_1^{n_1}}{n_1!}\frac{A_2^{n_2}}{n_2!}\cdots\frac{A_m^{n_1}}{n_m!} & W'\mathbf{n} \leq \mathbf{a} \\ 0 & otherwise \end{cases} \qquad (12)$$

where A_i is the traffic intensity to cell i. The constraint $W'\mathbf{n} \leq \mathbf{a}$ is a truncation of the state space which is a policy specific. For the fixed assignment it has a form $I\mathbf{n} \leq \mathbf{f}$, where f_i is a fixed number of channels assigned to cell i. For ordinary MP it has a form $U'\mathbf{n} \leq c\mathbf{e}$, where U is a matrix reflecting the reuse constraints and $\mathbf{e}$ is a vector, each element of which is unity. If the channels assigned to cells in groups of size b the space truncation is $U'\mathbf{r} \leq c\mathbf{e}$, where r_i is the total number of channels that must be allocated at cell i in order to carry n_i calls there.

Numerical calculation of blocking probabilities from the above equation is not so straightforward, especially if instead of possibility to use all c channels in cell, a maximal number of channels, called equipment limit, is used as a constraint. For a linear array of cells a simple iterative algorithm is possible, but for a general two-dimensional layout a Monte-Carlo technique is required. The authors demonstrated application of this technique to a less general form of dynamic assignment which allowed easy calculation of performance values. This was a mixed form between fixed and dynamic assignment, where division into channel groups was dynamically modified to adapt to the actual demand. Then the blocking probability for cell i in channel group Y can be calculated as:

$$\begin{aligned} B_{i:Y} = \sum \mathrm{Pr}(&\text{cell i in channel group } Y \text{ has n}_y \text{ calls}) \\ &\times \mathrm{Pr}(\text{all other cells in channel group } Y \text{ have } \leq n_y \text{ calls}) \\ &\qquad \times \mathrm{Pr}(\text{sum of channels used by other groups is } c \cdot n_y) \end{aligned}$$

3 CDMA traffic models

In difference from TDMA, CDMA is an interference-limited network and therefore its capacity is soft, it increases with relaxation of the QoS constraints. The users are separated by spreading codes with low cross-correlation, and the QoS is guaranteed if the received SNR is above a certain level defined by the system characteristics (modulation, coding gain, etc.). When the interference exceeds a maximal allowed level the user will be in outage, and the amount of interference in turn depends on the number of interferers and their link gains at the moment. Since the link gain changes with time due to user movements and changing channel conditions (fading), the CDMA instantaneous capacity may fluctuate for the same amount of users. The instantaneous capacity is determined by the system link gain matrix and is crucial for analysis of power and admission control algorithms. The teletraffic analysis usually deals with the average capacity determined by the interference distribution. The propagation conditions should be properly modelled to develop a realistic interference distribution.

First, a simple model neglecting propagation conditions from [Ev94] is presented. This model gives insight to CDMA teletraffic problems, it also uses the same product form solutions as for TDMA channel assignment in [Ev91]. Then, an example of a more realistic model for reverse link capacity calculation from [Vit95] is given. As an alternative the outage probability approximations from [Ev99] conclude the section.

3.1 Simple model with product form solution

This model addresses only the reverse (up) link, which is more analytically tractable than the forward (down) link. The main reason for that is that under perfect power control we can assume the power levels received at the base station to be equal for all the users in the cell. Then, the effective signal to noise ratio for L users using voice activity with factor α (fraction of time when the user is active) is:

$$\left(\frac{E_b}{N_0}\right)_{effective} = \frac{E_b}{N_0 + \alpha(L-1)E_b R/W} = \frac{E_b/N_0}{1 + (E_b/N_0)\alpha(L-1)/G} \tag{13}$$

where E_b is the energy per bit, W is the system bandwidth, R is the information rate, $G = W/R$ is the processing gain, N_0 is the thermal noise spectral density. To satisfy the QoS requirements $E_b/(N_0 + I_0)$ value should be more than a target value Γ.

The things are complicated by the inter-cell interference. Since, the BS only controls the power of its own users, the power received from the users in other cells is not equal to that of in-cell users. For the uniform user distribution and generic propagation conditions the propagation statistics does not differ between the cells, and the average intercell interference can be modelled by an inter-cell interference factor f. The value of this factor depends on propagation statistics as it will be shown in the following subsection. Assuming equal load between the cells, we can determine the maximal value for L to satisfy Γ by manipulating 13.

$$L(1+f) = \frac{G}{\alpha}\left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0}\right) + 1 \tag{14}$$

These equations describe a network with admission control based on load threshold, i.e. a new call is blocked when the number of users in the cell exceeds L. However, the real load in CDMA is determined by the total interference. Therefore, in case of unequal load, which is quite common in practice, we can admit more than L users if the level of inter-cell interference is low. Hence, admission control based on interference thresholds is more flexible. The authors of [Ev94] attempted to make a simple traffic model describing a network with interference-based admission by letting f be specific for a cell pair. Thus, f_{ij} describes the interference caused by the users in cell i to the users in cell j, and $f_{ii} = 0$. Then for M cells the admission control can allow only such a network state: $\{L_1, \ldots, L_M\}$ which fulfills the following condition:

$$\frac{E_b/N_0}{1 + (E_b/N_0)\alpha\left(L_i - 1 + \sum_{j=1}^{M} L_j f_{ij}\right)/G} \geq \Gamma \quad i = 1, \ldots, M \tag{15}$$

This inequality can be rewritten to obtain a constant on the right-hand side, which can be regarded as equivalent of the total capacity or effective number of channels for the network of M cells.

$$L_i + \sum_{j=1}^{M} L_j f_{ij} \leq \frac{G}{\alpha}\left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0}\right) + 1 \tag{16}$$

Now, L_i are random variables, representing the current number of calls in each cell. Note, that this model treats inter-cell interference as linear function of the number of users $I_{ij} = L_j f_{ij}$, whilch in reality is not true. This formulation is similar to the set of constraints in dynamic channel assignment from the previous section. Also in this case the probability distribution of the number of calls in each cell is given by a product form on a truncated state-space.

$$p(\mathbf{n}) = \begin{cases} p_o \prod_{i=1}^{M} \frac{A_i^{n_i}}{n_i!} & L_i - 1 + \sum_{j=1}^{M} L_j f_{ij} \leq c \quad \forall j \\ 0 & otherwise \end{cases} \tag{17}$$

where $c = \frac{G}{\alpha}\left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0}\right) + 1$.

This model was used to compare CDMA capacity to the capacity of the equivalent TDMA network with dynamic channel allocation. The comparison showed that CDMA does not provide any benefit if it has hard capacity constraints. However, by allowing soft capacity limits, reducing the value of Γ, the capacity of CDMA can be increased significantly at the cost of minor QoS degradation. However, it is not shown to what extent the QoS can be reduced. Admitting all call attempts we can achieve maximal utilisation but then we can not guarantee the required QoS. The ultimate capacity limit for CDMA is maximally allowed transmitted power, and the users who reached this limit would be in outage. Dropping users who had bad quality over some time would enable such a system to fully explore the soft capacity limits, but that would not comfort the dropped users and those who had bad quality during a large part of the call. In any case, this model was far from to represent a realistic system operation, but nevertheless it described the CDMA capacity in a generic and clear way.

3.2 Model for Erlang Capacity of Reverse Links

The next model [Vit95] was also developed for the same purpose of CDMA vs. TDMA capacity comparison. This comparison for the author, co-founder of Qualcomm, was not just of academic interest. May be that is why the model was more realistic. First, the propagation conditions were modelled to find a value of f for both hard and soft handoff cases. Here, we briefly present f calculation for the hard handoff case.

3.2.1 Inter-cell interference modelling

The propagation decay was modelled as the product of the m-th power of distance and a log-normal component representing shadowing losses. For a user at a distance r from a BS, decay is

$$D(r, \zeta) = r^m 10^{\zeta/10} \tag{18}$$

where ζ is the decibel attenuation due to shadowing, with zero mean and standard deviation σ. The random component of the decibel loss can be represented as a sum of two components: one in the near-field of the user, that is common to all BSs, and one in the near field of the BS, which is BS specific. Then, this component for the $i-$th BS is:

$$\zeta_i = a\xi + b\xi_{i,} \qquad \textit{where } a^2 + b^2 = 1, \quad a \leq 1 \tag{19}$$

with

$E(\zeta_i) = E(\xi) = E(\xi_{i,}) = 0,$
$V(\zeta_i) = V(\xi) = V(\xi_{i,}) = \sigma^2, \qquad \forall i$
$E(\xi\xi_{i,}) = 0, \qquad \forall i$

and

$E(\xi_i\xi_j), \qquad (i \neq j)$

Then, the normalised covariance of the losses to two BSs, i and j, is
$E(\xi_i\xi_j)/\sigma^2 = a^2 = 1 - b^2 \qquad (i \neq j)$
Further, $a^2 = b^2 = \frac{1}{2}$.

Under uniform user distribution assumption, for the normalised hexagonal cell radius and the average number of users per cell L, the user density $u = \frac{2L}{3\sqrt{3}}$.

For the user at coordinates (x, y) denote distance to the own BS $r_0(x, y)$ and to the neighbour BS , $r_1(x, y)$ and assuming the user's transmitter power equal to the propagation decay from 18 we can express the average inter-cell interference from a region S_0 to a given base station as:

$$I_{s_0} = E \iint\limits_{\bar{s}_0} \left[\frac{r_1^m(x, y)10^{\zeta_1/10}}{r_0^m(x, y)10^{\zeta_0/10}} \right] udA(x, y) \tag{20}$$

Here, the denominator represents the propagation decay to the given BS and numerator the gain adjustment by the neighbour's power control. Defining $R_1(x, y) = r_1(x, y)/r_0(x, y)$ and $\beta = \ln(10)/10$ we get:

$$I_{s_0} = Ee^{\beta(\zeta_1 - \zeta_0)} \iint\limits_{\bar{s}_0} R_1^m(x, y)udA(x, y) \tag{21}$$

From 19 $\zeta_1 - \zeta_0 = b(\xi_1 - \xi_0)$, which is a zero-mean Gaussian variable, and since ξ_1 and ξ_0 are independent, $V(\xi_1 - \xi_0) = 2\sigma^2$. Then, $Ee^{\beta(\zeta_1 - \zeta_0)}$ is equal to $e^{b^2(\beta\sigma)^2}$ and finally f can be expressed as:

$$f = \frac{I_{s_0}}{u} = e^{b^2(\beta\sigma)^2} \left[\frac{2}{3\sqrt{3}} \iint\limits_{\bar{s}_0} R_1^m(x, y)dA(x, y) \right] \tag{22}$$

This integral can be numerically integrated for different values of m and σ. For example, for $m = 4$ and $\sigma = 8$ $f = 2.38$. Inter-cell interference derivation for soft handoff is more complicated as it is more difficult to define regions S_0 and $\bar{S}_0$. Soft handoff considerably reduces inter-cell interference factor, for $m = 4$ and $\sigma = 8$ $f = 0.77$.

3.2.2 Reverse link capacity model

The total averaged power received by the cell with k_u perfectly controlled users with voice activity is:

$$P_r = \sum_{k=1}^{k_u} \nu_i E_b R + N_0 W, \tag{23}$$

where ν_i is a binary random variable indicating whether or not the i-th user is active at the instant, so that activity factor $\alpha = \Pr(\nu_i = 1) = 1 - \Pr(\nu_i = 0)$.

Since, P_r is the sum of noise, interference, and desired user power, then from 23 the averaged noise-plus-interference power denoted $I_0 W$ is:

$$I_0 W = \sum_{i=2}^{k_u} \nu_i E_b R + N_0 W \tag{24}$$

To prevent the system from overload, which characterises by exponential power growth, and thus to guarantee QoS, the interference-to-noise ratio should be limited:

$$\frac{I_0}{N_0} < \frac{1}{\eta}, \qquad \text{where } \eta < 1 \tag{25}$$

Combining this condition with 24 we get:

$$\sum_{i=2}^{k_u} \nu_i E_b R = (I_0 - N_0) W < I_0 W (1 - \eta) \tag{26}$$

or

$$\sum_{i=2}^{k_u} \nu_i < \frac{(W/R)(1-\eta)}{E_b/I_0} \triangleq K_0(1-\eta) \triangleq K_0^{'} \tag{27}$$

When this condition is not met the system is considered to be in outage. This condition is only temporary as both ν_i and k_u are random variables, and their variations would put the system forth and back from outage. The outage probability P_{out} is slightly upper bounded by inclusion ν_1 in the sum in 28.

$$P_{out} = \Pr(\sum_{i=2}^{k_u} \nu_i > K_0^{'}) < \Pr(\sum_{i=1}^{k_u} \nu_i > K_0^{'}) \tag{28}$$

The distribution of k_u is assumed to be described by:

$$P_k = \frac{A^k}{k!} e^{-A} \tag{29}$$

The author justifies this assumption by considering the LCH (last call held) system with call re-attempts. Then the distribution of the random variable $Z \triangleq \sum_{i=1}^{k_u} \nu_i$ is determined from its moment-generating function:

$$E(e^{sZ}) = \exp\left[A\alpha(e^s - 1\right] \tag{30}$$

This is the generating function of a Poisson distribution, and the outage probability is the sum of the Poisson tail:

$$P_{out} < e^{-A\alpha} \sum_{k=\lfloor K_0' \rfloor}^{\infty} \frac{(A\alpha)^k}{k!} \tag{31}$$

So far, this derivation was for a single cell case. For account for the multiple cells, k_u should be multiplied by $1 + f$, and correspondingly A has to be multiplied by $1 + f$ in 31.

For the tail of this distribution a Chernoff bound can be obtained:

$$\begin{aligned} P_{out} &< \min_{s>0} E\left\{\exp\left[s(Z - K_0')\right]\right\} = \\ & \exp\left\{-K_0'\left[\ln\left(\frac{K_0'}{A(1+f)\alpha}\right) - 1 + \frac{A(1+f)\alpha}{K_0'}\right]\right\} \end{aligned} \tag{32}$$

For large values of K_0', the Poisson distribution can be approximated by Gaussian with the same mean and variance, both being $A(1+f)\alpha$. Then,

$$P_{out} \approx \frac{1}{\sqrt{2\pi}} \int_{\frac{K_0' - A(1+f)\alpha}{\sqrt{A(1+f)\alpha}}}^{\infty} e^{-z^2/2} \tag{33}$$

The latter form fits quite well to the simulation results for large values of K_0' and it can be used to estimate capacity for given probability of outage and system parameters, while the Chernoff bound provides an upper bound and slightly underestimates capacity This model is also extended to the case with imperfect power control. The limitation of the model is the assumption on uniform user distribution under which the values of f were obtained. For the practical cases of network planning the capacity has to be estimated by taking into account the actual user distributions.

3.3 Approximations of Outage Probability from Interference Distributions

Also in [Ev99] approximations of outage probability are developed. The main difference from the Viterby approach is the network model and the definition of the outage probability. The network is modelled as a collection of independent $M/G/\infty$ queues. That means that there is no admission control, all the calls accepted and they can not be dropped. Also mobility and handover are not taken into account. Then, the network capacity is bounded by the outage probability constraint.

The outage probability is defined as probability for an arbitrary mobile anywhere in the network receives a reverse link SIR less than the required. If we assume that the received power for at a BS for a mobile is unity and I is a random variable representing the total BS received power then the outage probability is

$$B = P(1/I > \Gamma) = P(I > \Gamma) \tag{34}$$

Here Γ is defined as $\frac{W/R}{E_b/I_0}$ while in [Ev94] it was apparently $\frac{E_b}{N_0+I_0}$. For M cells with N_i calls in each I is given by:

$$I = \sum_{i=1}^{M} \sum_{j=1}^{N_i} I_{ij} \tag{35}$$

where I_{ij} is the interference from the j-th mobile to i-th cell. Further, let N be a Poisson random variable with parameter MA :

$$P(N = n) = e^{-MA} \frac{MA^n}{n!} \tag{36}$$

and I_j is a random variable with distribution function $M^{-1} \sum_{i=1}^{M} F_i$ being a finite mixture of original distribution functions. An example of a such distribution derivation is given in the paper. This derivation and resulting distribution are too long to be given here. Then, it can be shown by using characteristic functions that:

$$I \doteq \sum_{j=1}^{N} I_j \tag{37}$$

where symbol $\doteq$ means equality in distributions. Hence,

$$B = P(\sum_{j=1}^{N} I_j) > \Gamma \tag{38}$$

The outage probability was also approximated by the normal distribution. For the random variable $S_{N_A} = \sum_{j=1}^{N_A} X_j$, where N_A is a Poisson random variable with mean A, with mean $\mu_s = A\mu_X$ and $\sigma_s^2 = A(\sigma_x^2 + \mu_x^2)$ it was proven for both integer and real values of A that:

$$\frac{S_{N_A} - \mu_s}{\sigma_s} \Rightarrow \mathcal{N}(0,1) \qquad \text{as } A \to \infty \tag{39}$$

where $\Rightarrow$ means convergence in distribution and $\mathcal{N}(0,1)$ is a zero mean unit variance normal random variable.

An upper bound on the outage probability is also given by the Chernoff bound. The derivation is more formal and described in terms of large deviation theory. First, for the positive integer values of A the Poisson sum is rewritten as the deterministic sum $S_{N_A} \doteq \sum_{j=1}^{A} Y_i$ and the large deviation rate function is defined by

$$I(t) = \sup\,[\theta_t - M_Y(\theta)] \tag{40}$$

where θ is real and $M_Y(\theta) = \log E[e^{\theta Y_1}] = e^{M_x(\theta)} - 1$ is the log moment generating function (LMGF) of the $Y_{j.}$. Provided $M_Y(\theta) < \infty$ for all θ and that Y_1 is not bounded, $P(Y_1 \in (a,b)) < 1$ for all finite a and b, then from Cramer Theorem

$$\lim_{A\to\infty} \frac{1}{A} \log P(\frac{1}{A} S_{N_A} \geq \eta) = -I(\eta) \tag{41}$$

for $\eta > \mu_{Y.}$ Further, for any positive real value of A the Chernoff bound is :

$$\frac{1}{A}\log P(\frac{1}{A}S_{N_A} \geq \eta) \leq -I(\eta) \tag{42}$$

This bound can be extended to the case of real A either by modified use of Cramer's or by direct application of Gartner-Ellis Theorem. Introducing Γ to 42 we get:

$$\frac{1}{A}\log P(S_{N_A} \geq \Gamma) \leq -I(\frac{\Gamma}{A}) = \inf\left[M_Y(\theta) - \theta\frac{\Gamma}{A}\right] = \inf\left[e^{M_X(\theta)} - 1 - \theta\frac{\Gamma}{A}\right] \tag{43}$$

To validate the approximations the authors performed Monte Carlo simulations, generating a random number of users per cell at random locations, and compared them with both the Chernoff Bound and Gaussian approximation. For that they derived a complex interference distribution function for a simplified geometrical model, approximating the hexagonal cells by circles. The comparisons were done for different values of decay law index and Γ. For all cases the Chernoff bound overestimated outage probability by about an order of magnitude, underestimating capacity by 10-15 %. Normal approximation underestimates capacity for low traffic, especially for the law Γ case, since the Central Limit Theorem is not fully applicable for the small number of interferers. By using normal approximation and Chernoff bound the authors obtained well known results, like that the capacity for a fixed outage probability decreases as the decay law index decreases and increases with increase of Γ. The main value of this model is an attempt to directly map an interference distribution to the outage probability, though the used distribution function was rather complex.

4 Conclusions

The mobile networks are more challenging than the fixed networks to the teletraffic modelling due to variety of complex conditions which are difficult to model. In order to make analytically tractable models many assumptions and simplifications should be made. Because of that the traffic models, especially for CDMA networks, might not provide a direct solution to the network problems, so that network planning and control algorithms testing still rely on simulations. Nevertheless, the traffic models significantly contribute to understanding of network behaviour and capacity issues. The good models are those which find a trade-off between analytical tractability and realistic level of details. The Viterby's and Everitt's models of CDMA reverse links are examples of such models. However, there are still areas in mobile networks traffic modelling, e.g. CDMA downlink, where models of the same value were not yet developed.

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