# Switch Fabrics

**Switching Technology S38.165**

**http://www.netlab.hut.fi/opetus/s38165**

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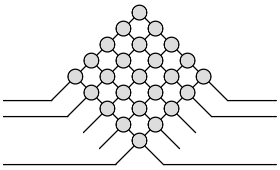
## Switch fabrics

- Basic concepts
- Time and space switching
- Two stage switches
- Three stage switches
- **Cost criteria**
- Multi-stage switches and path search

# Cost criteria for switch fabrics

- **Number of cross-points**
- **Fan-out**
- **Logical depth**
- **Blocking probability**
- **Complexity of switch control**
- **Total number of connection states**
- **Path search**



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# Cross-points

- **Number of cross-points gives the number of on-off gates (usually “and-gates”) in space switching equivalent of a fabric**
  - minimization of cross-point count is essential when cross-point technology is expensive (e.g. electro-mechanical and optical cross-points)
  - Very Large Scale Integration (VLSI) technology implements cross-point complexity in Integrated Circuits (ICs) => more relevant to minimize number of ICs than number of cross-points
  - Due to increasing switching speeds, large fabric constructions and increased integration density of ICs, power consumption has become a crucial design criteria
    - higher speed => more power
    - large fabrics => long buses, fan-out problem and more driving power
    - increased integration degree of ICs => heating problem

# Fan-out and logical depth

- VLSI chips can hide cross-point complexity, but introduce pin count and fan-out problem
  - length of interconnections between ICs can be long lowering switching speed and increasing power consumption
  - parallel processing of switched signals may be limited by the **number** of available **pins** of ICs
  - **fan-out** gives the driving capacity of a switching gate, i.e. number of inputs (gates/cross-points) that can be connected to an output
  - long buses connecting cross-points may lower the number of gates that can be connected to a bus
- Logical depth gives the number of cross-points a signal traverses on its way through a switch
  - large logical depth causes excessive delay and signal deterioration

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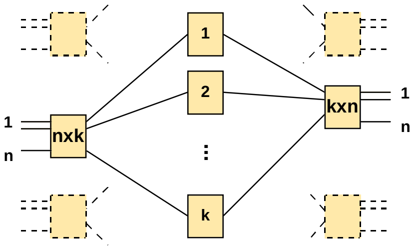
# Blocking probability

- Blocking probability of a multi-stage switching network difficult to determine
- Lee’s approximation gives a coarse measure of blocking
- Assume uniformly distributed load
  - equal load in each input
  - load distributed uniformly among intermediate stages (and their outputs) and among outputs of the switch
- Probability that an input is engaged is $a = \lambda S$ where
  - $\lambda$ = input rate on an input link
  - $S$ = average holding time of a link



# Blocking probability (cont.)

- Under the assumption of uniformly distributed load, probability that a path between any two switching blocks is engaged is $p = an/k$ ($k \geq n$)
- Probability that a certain path from an input block to an output block is engaged is $1 - (1-p)^2$ where the last term is the probability that both (input and output) links are disengaged
- Probability that all $k$ paths between an input switching block and an output switching block are engaged is

$$B = [1 - (1- an/k)^2]^k$$

which is known as Lee's approximation

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# Control complexity

- Give a graph ***G*** , a control algorithm is needed to find and set up paths in ***G*** to fulfill connection requirements
- Control complexity is defined by the hardware (computation and memory) requirements and the run time of the algorithm
- Amount of computation depends on blocking category and degree of blocking tolerated
- In general, computation complexity grows exponentially as a function of the number of terminal
- There are interconnection networks that have a regular structure for which control complexity is substantially reduced
- There are also structures that can be distributed over a large number of control units

# Management complexity

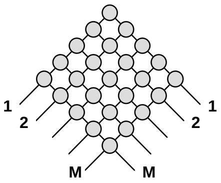
- **Network management** involves adaptation and maintenance of a switching network after the switching system has been put in place
- Network management deals with
  - failure events and growth in connectivity demand
  - changes of traffic patterns from day to day
  - overload situations
  - diagnosis of hardware failures in switching system, control system as well as in access and trunk network
    - in case of failure, traffic is rerouted through redundant built-in hardware or via other switching facilities
    - diagnosis and failure maintenance constitute a significant part of software of a switching system
  - In order for switching cost to grow linearly in respect to total traffic, switching functions (such as control, maintenance, call processing and interconnection network) should be as modular as possible

# Example 1

- A switch with
  - a capacity of *N* simultaneous calls
  - average occupancy on lines during busy hour is ***X*** Erlangs
  - ***Y*** % requirement for internal use
  - notice that two (one-way) connections are needed for a call

  requires a switch fabric with ***M*** = 2 x [(100+***Y***)/100] x(***N***/***X***) inputs and outputs.
- If *N* = 20 000, *X* = 0.72 and *Y* = 10%

  => *M* = 2 x 1.1 x 20 000/0.72 = 61 112
  => corresponds to 2038 E1 links



# Amount of traffic in Erlangs

- Erlang defines the amount of traffic flowing through a communication system - it is given as the aggregate holding time of all channels of a system divided by the observation time period
- Example 1:
  - During an hour period three calls are made (5 min, 15 min and 10 min) using a single telephone channel => the amount of traffic carried by this channel is (30 min/60 min) = 0.5 Erlang
- Example 2:
  - a telephone exchange supports 1000 channels and during a busy hour (10.00 - 11.00) each channel is occupied 45 minutes on the average => the amount of traffic carried through the switch during the busy hour is (1000x45 min / 60 min) = 75 Erlangs

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# Erlang's first formula

**Erlang 1st formula**

$$E_1(n, A) = \frac{\frac{A^n}{n!}}{1 + A + \frac{A^2}{2!} + \cdots + \frac{A^n}{n!}}$$

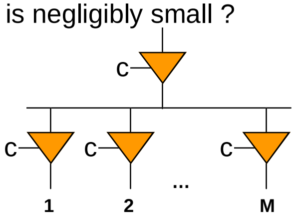
- Erlang 1st formula applies to systems fulfilling conditions
  - a failed call is disconnected (loss system)
  - full accessibility
  - time between subsequent calls vary randomly
  - large number of sources
- $E_1$(5, 2.7) implies that we have a system of 5 inlets and offered load is 2.7 Erlangs - blocking calculated using the formula is 8.5 %
- Tables and diagrams (based on Erlang's formula) have been produced to simplify blocking calculations

## Example 2

- An exchange for 2000 subscribers is to be installed and it is required that the blocking probability should be below 10 %. If E2 links are used to carry the subscriber traffic to telephone network, how many E2 links are needed ?
  - average call lasts 6 min
  - a subscriber places one call during a 2-hour busy period (on the average)
- Amount of offered traffic is (2000x6 min /2x60 min) = 100 Erl.
- Erlang 1st formula gives for 10 % blocking and load of 100 Erl. that *n* = 97
  => required number of E1 links is ceil(97/30) = 4

## Example 3

- Suppose driving current of a switching gate (cross-point) is 100 mA and its maximum input current is 8 mA
- How many output gates can be connected to a bus, driven by one input gate, if the capacitive load of the bus is negligibly small ?
- Fan-out = floor[100/8] = 12



- How many output gates can be connected to a bus driven by one input gate if load of the bus corresponds to 15 % of the load of a gate input) ?
- Fan-out = floor[100/(1.15x8)] = 10

# Switch fabrics

- Basic concepts
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- Two stage switches
- Three stage switches
- Cost criteria
- **Multi-stage switches and path search**

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# Multi-stage switching

- Large switch fabrics could be constructed by using a single *N*x*N* crossbar, interconnecting *N* inputs to *N* outputs
  - such an array would require $N^2$ cross-points
  - logical depth = 1
  - considering the limited driving power of electronic or optical switching gates, large *N* means problems with signal quality (e.g. delay, deterioration)
- Multi-stage structures can be used to avoid above problems
- Major design problems with multi-stages
  - find a non-blocking structure
  - find non-conflicting paths through the switching network

# Multi-stage switching (cont.)

- Let’s take a network of $K$ stages
- Stage $k$ ($1 \le k \le K$) has $r_k$ **switch blocks** (**SB**)
- Switch block $j$ ($1 \le j \le r_k$) in stage $k$ is denoted by $S(j,k)$
- Switch $j$ has $m_k$ inputs and $n_k$ outputs
- Input $i$ of $S(j,k)$ is represented by $e(i,j,k)$
- Output $i$ of $S(j,k)$ is represented by $o(i,j,k)$
- Relation $o(i,j,k)= e(i’,j’,k+1)$ gives interconnection between output $i$ and input $i’$ of switch blocks $j$ and $j’$ in consecutive stages $k$ and $k$+1
- Special class of switches:
  - $n_k = r_{k+1}$ and $m_k = r_{k-1}$
  - each SB in each stage connected to each SB in the next stage

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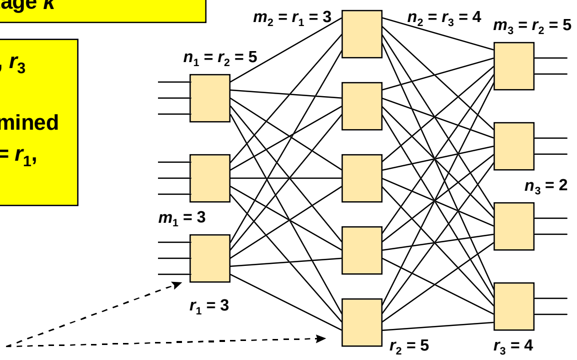
# Clos network

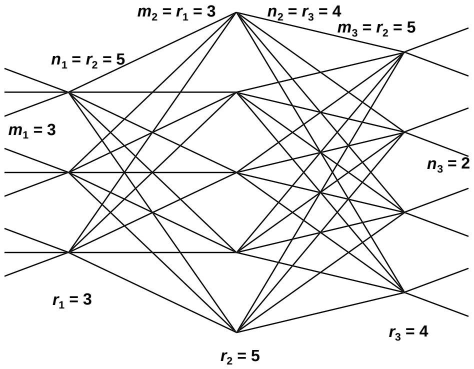
**$m_k$ = number of inputs in a SB at stage $k$**
**$n_k$ = number of outputs in a SB at stage $k$**
**$r_k$ = number of SBs at stage $k$**

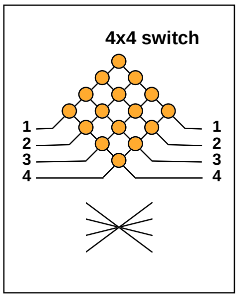
- **parameter $m_1$, $n_3$, $r_1$, $r_2$, $r_3$ chosen freely**
- **other parameters determined uniquely by $n_1 = r_2$, $m_2 = r_1$, $n_2 = r_3$, $m_3 = r_2$**



**SB = Switch Block**

# Graph presentation of a Clos network





**Every SB in stage *k* is connected to all $r_{k+1}$ SBs in the following stage *k*+1 with a single link.**

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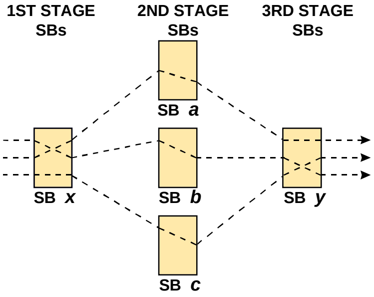
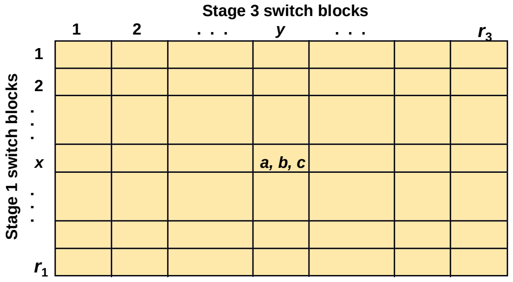
# Path connections in a 3-stage network

- An input of SB ***x*** may be connected to an output of SB ***y*** via a middle stage SB ***a***
- Other inputs of SB ***x*** may be connected to other outputs of SB ***y*** via other middle stage SBs (***b***, ***c***, …)
- Paull's connection matrix is used to represent paths in three stage switches



# Paull’s matrix

- Middle stage switch blocks (***a***, ***b***, ***c***) connecting 1st stage SB ***x*** to 3rd stage SB ***y*** are entered into entry (***x***,***y***) in $r_1$ x $r_3$ matrix
- Each entry of the matrix may have 0, 1 or several middle stage SBs
- A symbol (***a***,***b***,..) appears as many times in the matrix as there are connections through it



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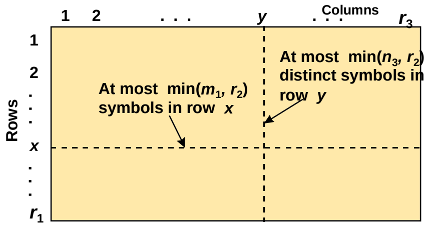
# Paull’s matrix (cont.)

## Conditions for a legitimate point-to-point connection matrix:

1. Each row has at most $m_1$ symbols, since there can be as many paths through a 1st stage SB as there are inputs to it
2. Each column has at most $n_3$ symbols, since there can be as many paths through a 3rd stage SB as there are outputs from it



# Paull’s matrix (cont.)

## Conditions of a legitimate point-to-point connection matrix (cont.):

3 Symbols in each row must be distinct, since only one edge connects a 1st stage SB to a 2nd stage SB
=> there can be at most $r_2$ different symbols

4 Symbols in each column must be distinct, since only one edge connects a 2nd stage SB to a 3rd stage SB and an edge does not carry signals from several inputs
=> there can be at most $r_2$ different symbols

**In case of multi-casting, conditions 1 and 3 may not be valid, because a path from the 1st stage may be directed via several 2nd stage switch blocks. Conditions 2 and 4 remain valid.**

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# Strict-sense non-blocking Clos

## Definitions:

- *T’* is a subset of set *T* of transmitting terminals
- *R’* is a subset of set *R* of receiving terminals
- Each element of *T’* is connected by a legitimate multi-cast tree to a non-empty and disjoint subset *R’*
- Each element of *R’* is connected to one element of *T’*

A network is **strict sense non-blocking** if any $t \in T - T'$ can establish a legitimate multi-cast tree to any subset $R - R'$ without changes to the previously established paths.

A **rearrangeable** network satisfies the same conditions, but allows changes to be made to the previously established paths.

# Clos theorem

## Clos theorem:

A Clos network is **strict-sense non-blocking** if and only if the number of 2nd stage switch blocks fulfills the condition

$$r_2 \geq m_1 + n_3 - 1$$

- A symmetric Clos network with $m_1 = n_3 = n$ is strict-sense non-blocking if

$$r_2 \geq 2n - 1$$

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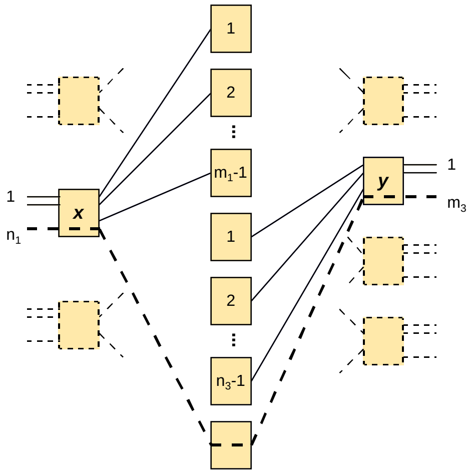
# Proof of Clos theorem

## Proof 1:

- Let's take some SB ***x*** in the 1st stage and some SB ***y*** in the 3rd stage, which both have maximum number of connection minus one.  
  => ***x*** has $m_1$ -1 and ***y*** has $n_3$ -1 connections
- One additional connection should be established between ***x*** and ***y***
- In the worst case, existing connections of ***x*** and ***y*** occupy distinct 2nd stage SBs  
  => $m_1$ -1 SBs for paths of ***x*** has and $n_3$ -1 SBs for paths of ***y***
- To have a connection between ***x*** and ***y*** an additional SB is needed in the 2nd stage  
  => required number of SBs is $(m_1 -1) + (n_3 -1) + 1 = m_1 + n_3 -1$

# Visualization of proof



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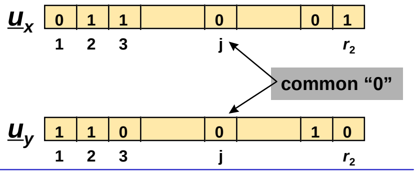
# Paull’s matrix and proof of Clos theorem

## Proof 2:

- A connection from an idle input of a 1st stage SB ***x*** to an idle output of a 3rd stage SB ***y*** should be established
- $m_1$-1 symbols can exist already in row ***x***, because there are $m_1$ inputs to SB ***x***.
- $n_3$-1 symbols can exist already in row ***y***, because there are $n_3$ outputs to SB ***y***.
- In the worst case, all the ($m_1$-1 + $n_3$-1) symbol are distinct
- To have an additional path between ***x*** and ***y***, one more SB is needed in the 2nd stage
  => $m_1 + n_3 - 1$ SBs are needed

# Procedure for making connections

- Keep track of symbols used by row ***x*** using an occupancy vector $\underline{u}_x$ (which has $r_2$ entries that represent SBs of the 2nd stage)
- Enter “1” for a symbol in $\underline{u}_x$ if it has been used in row ***x***, otherwise enter “0”
- Likewise keep track of symbols used by column ***y*** using an occupancy vector $\underline{u}_y$
- To set up a connection between SB ***x*** and SB ***y*** look for a position ***j*** in $\underline{u}_x$ and $\underline{u}_y$ which has “0” in both vectors
- Amount of required computation is proportional to $r_2$



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# Rearrangeable networks

## Slepian-Duguid theorem:

A three stage network is rearrangeable if and only if

$$r_2 \geq \max(m_1, n_3)$$

A symmetric Clos network with $m_1 = n_3 = n$ is rearrangeably non-blocking if

$$r_2 \geq n$$

## Paull’s theorem:

The number of circuits that need to be rearranged is at most

$$\min(r_1, r_3) -1$$

# Connection rearrangement by Paull's matrix

- If there is no common symbol (position *j*) found in $\underline{u}_x$ and $\underline{u}_y$, we look for symbols in $\underline{u}_x$ that are not in $\underline{u}_y$ and symbols in $\underline{u}_y$ not found in $\underline{u}_x$ => a new connection can be set up only by rearrangement
- Let's suppose there is symbol ***a*** in $\underline{u}_x$ (not in $\underline{u}_y$) and symbol ***b*** in $\underline{u}_y$ (not in $\underline{u}_x$) and let's choose either one as a starting point
- Let it be ***a*** then ***b*** is searched from the column in which ***a*** resides (in row ***x***) - let it be column $j_1$ in which ***b*** is found in row $i_1$
- In row $i_1$ search for ***a*** - let this position be column $j_2$ n
- This procedure continues until symbol ***a*** or ***b*** cannot be found in the column or row visited

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\underline{u}_x$ | 1 | 1 | | 0 | | 1 | | 1 | 1 |
| | 1 | 2 | | a | | b | | | $r_2$ |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\underline{u}_y$ | 1 | 1 | | 1 | | 0 | | 1 | 1 |
| | 1 | 2 | | a | | b | | | $r_2$ |

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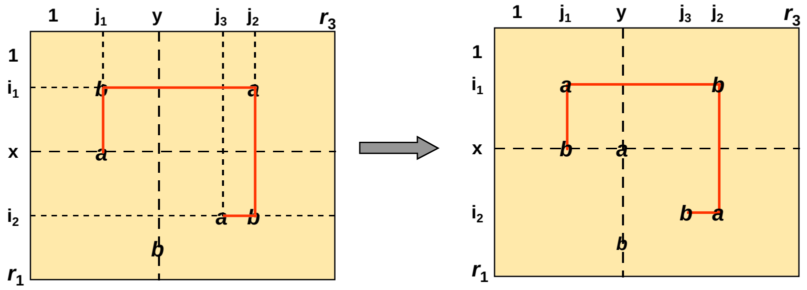
# Connection rearrangement by Paull's matrix (cont.)

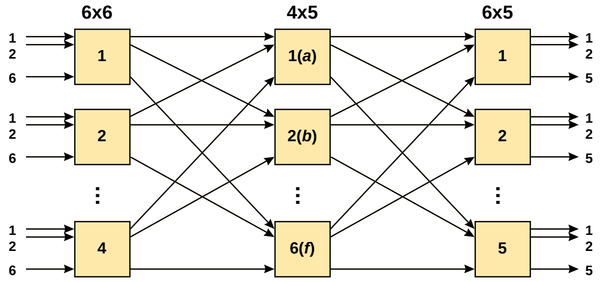
- At this point connections identified can be rearranged by replacing symbol ***a*** (in rows ***x***, $i_1$, $i_2$, ...) by ***b*** and symbol ***b*** (in columns ***y***, $j_1$, $j_2$, ...) by ***a***
- ***a*** and ***b*** still appear at most once in any row or column
- 2nd stage SB ***a*** can be used to connect ***x*** and ***y***



# Example of connection rearrangement by Paull's matrix

- Let's take a three-stage network 24x25 with $r_1$=4 and $r_3$=5
- Rearrangeability condition requires that $r_2$=6
  - let these SBs be marked by *a*, *b*, *c*, *d*, *e* and *f*

=> $m_1 = 6$, $n_1 = 6$, $m_2 = 4$, $n_2 = 5$, $m_3 = 6$, $n_3 = 5$



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# Example of connection rearrangement by Paull's matrix (cont.)

- In the network state shown below, a new connection is to be established between SB1 of stage 1 and SB1 of stage 3
- No SBs available in stage 2 to allow a new connection
- Slepian-Duguid theorem => a three stage network is rearrangeable if and only if $r_2 \geq \max(m_1, n_3)$
  - $m_1 = 6$, $n_3 = 5$, $r_2 = 6$ => condition fulfilled
- SBs ***c*** and ***d*** are selected to operate rearrangement

3rd stage SBs

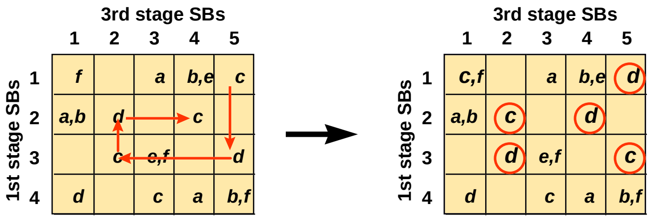
| 1st stage SBs | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | *f* | | *a* | *b,e* | *c* |
| 2 | *a,b* | *d* | | *c* | |
| 3 | | *c* | *e,f* | | *d* |
| 4 | *d* | *c* | *a* | | *b,f* |

**Occupancy vectors of SB1/stage 1 and SB1/stage 3**

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| $\underline{u}_{1\text{-}1}$ | 1 | 1 | 1 | 0 | 1 | 1 |
| $\underline{u}_{3\text{-}1}$ | 1 | 1 | 0 | 1 | 0 | 0 |

# Example of connection rearrangement by Paull's matrix (cont.)

- Start rearrangement procedure from symbol ***c*** in row 1 and column 5
- 5 connection rearrangements are needed to set up the required connection - Paull's theorem !!!



3rd stage SBs

1st stage SBs

|  | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | *f* |  | *a* | *b,e* | *c* |
| 2 | *a,b* | *d* |  | *c* |  |
| 3 |  | *c* | *e,f* |  | *d* |
| 4 | *d* |  | *c* | *a* | *b,f* |

→

3rd stage SBs

1st stage SBs

|  | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | *c,f* |  | *a* | *b,e* | *d* |
| 2 | *a,b* | *c* |  | *d* |  |
| 3 |  | *d* | *e,f* |  | *c* |
| 4 | *d* |  | *c* | *a* | *b,f* |

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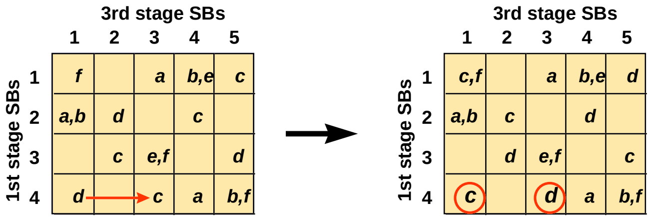
# Example of connection rearrangement by Paull's matrix (cont.)

- Paull's theorem states that the number of circuits that need to be rearranged is at most min($r_1$, $r_3$) -1 = 3
  => there must be another solution
- Start rearrangement procedure from ***d*** in row 4 and column 1
  => only one connection rearrangement is needed



3rd stage SBs

1st stage SBs

|  | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | *f* |  | *a* | *b,e* | *c* |
| 2 | *a,b* | *d* |  | *c* |  |
| 3 |  | *c* | *e,f* |  | *d* |
| 4 | *d* |  | *c* | *a* | *b,f* |

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3rd stage SBs

1st stage SBs

|  | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | *c,f* |  | *a* | *b,e* | *d* |
| 2 | *a,b* | *c* |  | *d* |  |
| 3 |  | *d* | *e,f* |  | *c* |
| 4 | *c* |  | *d* | *a* | *b,f* |

# Recursive construction of switching networks

- To reduce cross-point complexity of three stage switches individual stages can be factored further
- Suppose we want to construct an *NxN* switching network and let *N* = *pxq*
- A rearrangeably non-blocking Clos network is constructed recursively by connecting a *pxp*, *qxq* and *pxp* rearrangeably non-blocking switch together in respective order
  => under certain conditions result may be a strict-sense non-blocking network
- A strict-sense non-blocking network is constructed recursively by connecting a $p(2p - 1)$, *qxq* and $p(2p - 1)$ strict-sense non-blocking switch together in respective order
  => result may be a rearrangeable non-blocking network

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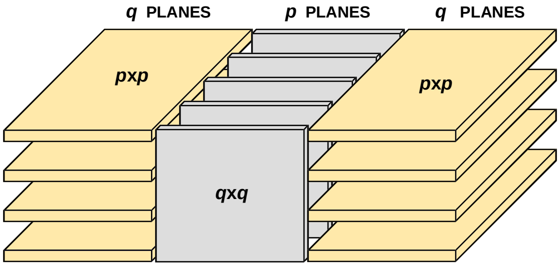
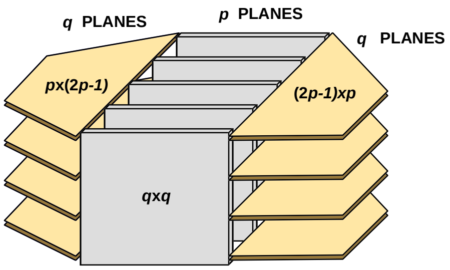
# 3-dimensional construction of a rearrangeably non-blocking network



Number of cross-points for the rearrangable construction is

$$p^2q + q^2p + p^2q = 2\,p^2q + q^2p$$

# 3-dimensional construction of a strict-sense non-blocking network



Number of cross-points for the strictly non-blocking construction is

$$p(2p - 1)q + q^2 (2p - 1) + p (2p - 1)q = 2p(2p - 1) q + q^2 (2p - 1)$$

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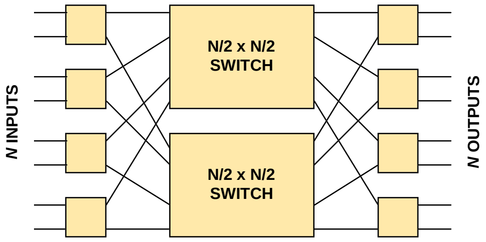
# Recursive factoring of switching networks

- *N* can be factored into *p* and *q* in many ways and these can be factored further
- Which *p* to choose and how should the sub-networks be factored further ?
- Doubling in the 1st and 3rd stages suggests to start with the smallest factor and recursively factor $q = N/p$ using the next smallest factor  
=> this strategy works well for rearrangeable networks  
=> for strict-sense non-blocking networks width of the network is doubled  
=> not the best strategy for minimizing cross-point count
- Ideal solution: low complexity, minimum number of cross-points and easy to construct => quite often conflicting goals

# Recursive factoring of a rearrangeably non-blocking network

- Special case $N = 2^n$, $n$ being a positive integer
  => a **rearrangeable network** can be constructed by factoring $N$ into
  $p = 2$ and $q = N/2$
  => resulting network is a Benes network
  => each stage consists of $N/2$ switch blocks of size 2x2
- Factor $q$ relates to the multiplexing factor (number of time-slots on inputs)
  => recursion continued until speed of signals low enough for real implementations



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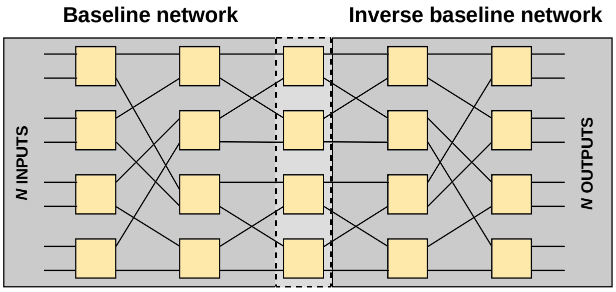
# Benes network



Number of stages in a Benes network

$$K = 2\log_2 N - 1$$

# Benes network (cont.)

- Benes network is recursively constructed of 2x2 switch blocks and it is rearrangeably non-blocking (see Clos theorem)
- First half of Benes network is called baseline network
- Second half of Benes network is a mirror image (inverse) of the first half and is called inverse baseline network
- Number of switch stages is $K = 2\log_2 N - 1$
- Each stage includes *N*/2 2x2 switching blocks (SBs) and thus number of SBs of a Benes network is

$$N\log_2 N - (N/2) = N(\log_2 N - ½)$$

- Each 2x2 SB has 4 cross-points and number of cross-points in a Benes network is

$$4(N/2)(2\log_2 N-1) = 4N\log_2 N - 2N \sim \quad 4N\log_2 N$$

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# Illustration of recursively factored Benes network