HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 1**
Networking Laboratory | 9.2.2004
S-38.215 Special Course in Networking Technology, Spring 2004 | Aalto

1. (Lawler, Exercise 1.2) Consider a Markov chain with state space $\{0, 1\}$ and transition matrix

$$P = \begin{bmatrix} 1/3 & 2/3 \\ 3/4 & 1/4 \end{bmatrix}$$

Assuming that the chain starts in state 0 at time $n = 0$, what is the probability that it is in state 1 at time $n = 3$?

2. (Lawler, Exercise 1.5) Consider the Markov chain with state space $S = \{0, \ldots, 5\}$ and transition matrix

$$P = \begin{bmatrix} .5 & .5 & 0 & 0 & 0 & 0 \\ .3 & .7 & 0 & 0 & 0 & 0 \\ 0 & 0 & .1 & 0 & .9 & 0 \\ .25 & .25 & 0 & 0 & .25 & .25 \\ 0 & 0 & .7 & 0 & .3 & 0 \\ 0 & .2 & 0 & .2 & .2 & .4 \end{bmatrix}.$$

What are the communication classes? Which ones are recurrent and which are transient? Suppose the system starts in state 0. What is the probability that it will be in state 0 at some large time? Answer the same question assuming that the system starts in state 5.

3. Consider the following random walk with reflecting boundaries:

$$\begin{aligned} &p(0, 1) = 1, \\ &p(i, i-1) = p(i, i+1) = 1/2, \quad \text{for all } i = 1, \ldots, N-1, \\ &p(N, N-1) = 1. \end{aligned}$$

What is the period of this irreducible Markov chain? Find the invariant distribution. Find the following limits for all $i = 0, \ldots, N$:

$$\lim_{n\to\infty} p_{2n}(0, i) \quad \text{and} \quad \lim_{n\to\infty} p_{2n+1}(0, i).$$