1. (Lawler, Exercise 2.3) Consider the Markov chain with state space $S = \{0, 1, 2, \ldots\}$ and transition probabilities

$$p(x, x+1) = 2/3, \quad p(x, 0) = 1/3.$$

   Show that the chain is positive recurrent and give the limiting probability distribution $\bar{\pi}$.

2. (Lawler, Exercise 2.5) Let X_n be a Markov chain with state space $S = \{0, 1, 2, \ldots\}$. For each of the following transition probabilities, state if the chain is positive recurrent, null recurrent, or transient. If it is positive recurrent, give the stationary probability distribution:

   (a) $p(x, 0) = 1/(x+2), \quad p(x, x+1) = (x+1)/(x+2)$;

   (b) $p(x, 0) = (x+1)/(x+2), \quad p(x, x+1) = 1/(x+2)$;

   (c) $p(x, 0) = 1/(x^2+2), \quad p(x, x+1) = (x^2+1)/(x^2+2)$.

3. Let A be the infinitesimal generator for an irreducible, finite-state, continuous-time Markov chain. Let $\bar{\pi}$ denote the invariant distribution of this chain, i.e.

$$\bar{\pi} A = 0.$$

   Construct such a transition matrix P for which $\bar{\pi}$ (given above) is the invariant distribution, i.e.

$$\bar{\pi} P = \bar{\pi}.$$

   (*Hint*: See the exercises in Lawler, Section 3.5.)