1. Construct such an irreducible, positive recurrent birth-and-death process X_t for which the corresponding discrete-time Markov chain J_n embedded in the jump times is null recurrent.

2. (Lawler, Exercise 3.7) Consider the continuous-time Markov chain with state space $\{1, 2, 3, 4\}$ and infinitesimal generator

$$A = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

   (a) Find the equilibrium distribution $\bar{\pi}$.

   (b) Suppose the chain starts in state 1. What is the expected time until it changes state for the first time?

   (c) Again assume the chain starts in state 1. What is the expected time until the chain is in state 4?

3. (Lawler, Exercise 3.8) Let X_t be a continuous-time birth-and-death process with birth rate $\lambda_n = 1 + (1/(n+1))$ and death rate $\mu_n = 1$. Is this process positive recurrent, null recurrent, or transient? What if $\lambda_n = 1 - (1/(n+2))$?