HELSINKI UNIVERSITY OF TECHNOLOGY **Exercise 6**
Networking Laboratory 15.3.2004
S-38.215 Special Course in Networking Technology, Spring 2004 Aalto

1. (Lawler, Exercise 4.7) Consider a simple "Wheel of Fortune" game. A wheel is divided into 12 equal-sized wedges. Eleven of the wedges are marked with the numbers $100, 200, \ldots, 1100$ denoting an amount of money won if the wheel lands on those numbers. The twelfth wedge is marked "bankrupt." A player can spin as many times as he or she wants. Each time the wheel lands on a numbered wedge, the player receives that much money which is added to his/her previous winnings. However, if the wheel ever lands on the "bankrupt" wedge, the player lose all of his/her money that has been won up to that point and the game stops. The player may quit at any time, and take all the money he or she has won (assuming that the "bankrupt" wedge has not come up). Assuming that the goal is to maximize one's expected winnings in this game, devise an optimal strategy for playing this game and compute one's expected winnings.

2. (Lawler, Exercise 5.1) Consider the experiment of rolling two dice, X_1 and X_2. Find the conditional expectation $E[X_1|X_1 + X_2]$.

3. Consider a discrete-time random walk X_n in finite state space $\{0, 1, \ldots, N\}$ with transition probabilities

$$\begin{aligned}
p(0,j) &= \begin{cases} q, & j = 1, \\ 1-q, & j = 0; \end{cases} \\
p(i,j) &= \begin{cases} p, & 0 < i < N, \quad j = i+1, \\ 1-p, & 0 < i < N, \quad j = i-1; \end{cases} \\
p(N,j) &= \begin{cases} r, & j = N, \\ 1-r, & j = N-1; \end{cases}
\end{aligned}$$

With which values of q, p, and r, random walk X_n is a martingale with respect to its own history $\sigma(X_0, \ldots, X_n)$?