HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 7**
Networking Laboratory | 22.3.2004
S-38.215 Special Course in Networking Technology, Spring 2004 | Aalto

1. Let X be a random variable. Fix $n > 0$ and prove that

$$P(A) \leq P\{|X| > n\} \quad \Rightarrow \quad E[|X|I_A] \leq E[|X|I_{\{|X|>n\}}],$$

where I_A refers to the indicator of event A.

2. (Lawler, Exercise 5.6 (a)) Consider a biased random walk on the integers with probability $p < 1/2$ of moving to the right and probability $1-p$ of moving to the left. Let S_n be the value at time n and assume that $S_0 = a$, where $0 < a < N$. Show that $M_n = ((1-p)/p)^{S_n}$ is a martingale.

3. (Lawler, Exercise 5.6 (b)) Let S_n and M_n be as in Problem 2. Let T be the first time that the random walk reaches 0 or N, i.e.,

$$T = \min\{n \mid S_n \in \{0, N\}\}.$$

(a) Verify the conditions of the optional sampling theorem for martingale M_n and stopping time T.

(b) Use optional sampling on the martingale M_n to compute $P\{S_T = 0\}$.