

1. (Lawler, Exercise 8.6) Suppose  $X_t$  is a standard Brownian motion and  $Y_t = a^{-1/2}X_{at}$  with  $a > 0$ . Show that  $Y_t$  is a standard Brownian motion.
2. (Lawler, Exercise 8.7) Suppose  $X_t$  is a standard Brownian motion and  $Y_t = tX_{1/t}$ . Show that  $Y_t$  is a standard Brownian motion. (*Hint:* If  $U$  and  $V$  are normally distributed, then a sufficient and necessary condition for their independence is  $E[UV] = E[U]E[V]$ .)
3. (Lawler, Exercise 8.8) Let  $X_t$  be a standard Brownian motion. Compute the following conditional probability:

$$P\{X_2 > 0 \mid X_1 > 0\}.$$

Are the events  $\{X_1 > 0\}$  and  $\{X_2 > 0\}$  independent?