HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.3141 Teletraffic Theory, IV/2008

Exercise 1
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1. Traffic measurements of a telephone exchange on five consecutive days gave the following data for the average number of calls in progress during different hours:

Hour	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17
Day 1	71	90	95	102	93	89	95	98	87	82
Day 2	78	98	106	99	101	92	93	100	94	87
Day 3	75	93	98	103	107	100	98	98	92	93
Day 4	74	95	96	104	97	95	95	101	99	100
Day 5	79	94	96	101	100	99	92	97	102	98

   Determine the busy hour traffic according to different definitions: a) ADPH, b) TCBH c) FDMH (between 10-11). In all the cases a five days average is to be used.

2. A computer shop has 10 portable PCs available for rent. The average rental time is 2.5 days and is exponentially distributed. Customers arrive randomly at an average rate of five per day. If a PC is not available, a customer will go away without returning.

   a) What fraction of the arriving customers will be lost?

   b) What is the average number of PCs on rent?

   c) What is the probability a PC on rent will be on rent for more than five days?

   d) The profit on a computer rented is 20 euros per day. One of the PCs is destroyed on an accident. What is the expected loss of profit per day?

3. Calls arrive at a system with infinitely many lines according to a Poisson process with unknown traffic intensity a. Assume that all values of a, $a \in [0, \infty)$ are equally likely a priori. Observations on the system occupancy $n_1,\ n_2, \ldots$ are made at time instants that are sufficiently far apart to consider the observations to be independent. Using the Bayes formula, determine the posterior distribution of a after the observation n_1. Then (using the previously obtained distribution as a priori) determine the posterior distribution after the observation n_2 and generally after the observations $n_1,\ n_2, \ldots, n_k$. What is the maximum and the standard deviation of this distribution?

4. Consider the sum that appears in the denominator of the Erlang blocking formula $E(n, a)$,

$$G(n, a) = 1 + \frac{a}{1!} + \frac{a^2}{2!} + \ldots + \frac{a^n}{n!}$$

   a) Show that the blocking formula $E(n, a)$ can be written as

$$E(n, a) = 1 - \frac{d}{da} \log G(n, a)$$

   b) Show by integration by parts that for integer n it holds

$$G(n, a) = \frac{e^a}{n!} \int_a^\infty t^n e^{-t} dt$$

5. For purposes of this exercise, consider a trivial overflow system consisting of one trunk primary system and one trunk secondary system. Otherwise, standards assumptions are made: Poisson arrivals (λ) and exponential holding time (μ).

   Draw the state diagram of the system, defining both the occupancy of the primary group (trunk) and that of the secondary group (trunk), and solve the equilibrium probabilities.

   Clarify the relation of this problem to the loss system consisting of two trunks. Determine the time and call blocking probabilities a) of the overflow trunk b) of the whole system.

6. An Erlang loss system consisting of five trunks receives an arriving traffic stream of intensity $a = 5$ Erl. The call that are blocked are offered to be carried on an overflow trunk group. a) What is the intensity of the overflow traffic? b) Which fraction of the overflow traffic is blocked in the overflow trunk group? Hint: The original trunk group and the overflow trunk group together form a loss system with six trunks. c) What would be the blocking probability on the overflow trunk group if the overflow traffic offered to it were Poissonian (whence the blocking can be calculated with Erlang formula).