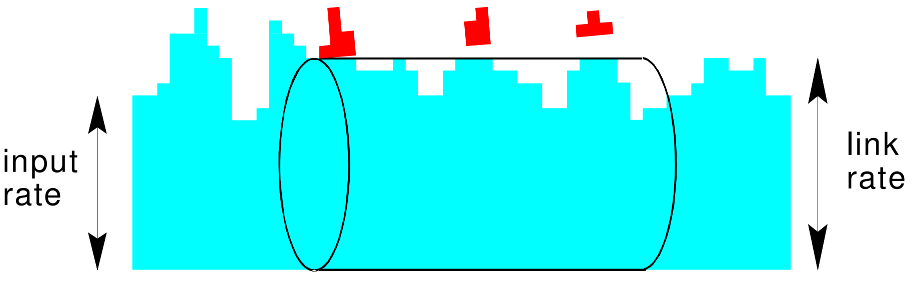
BURST LEVEL BEHAVIOUR

Overflow probability in a bufferless system

- Consider an output port of an ATM switch
- Suppose:
  - the buffer can only absorb cell level fluctuations
  - the capacity of the outgoing link is C
- Several VC connections use the same outgoing link
  - denote the total bitrate of the connections by X
- The rate exceeding C flows over
- The overflow rate $(X - C)^+$
  - where $(\cdot)^+ = \max(0, \cdot)$



Overflow and saturation probabilities

- Overflow probability
  - the ratio of the overflow rate to the arrival rate

$$P_{\text{loss}} = \frac{\mathrm{E}[(X-C)^+]}{\mathrm{E}[X]} = \frac{\int_C^\infty (x-C)f(x)dx}{\int_0^\infty xf(x)dx}$$

- Saturation probability

$$P_{\text{sat}} = \mathrm{P}\{X \geq C\}$$

  - probability that the capacity is exceeded
  - sometimes easier to evaluate than P_{loss}
  - rough upper bound for P_{loss} (not strictly a bound, though)
  - usually a few orders (factor 100) higher than P_{loss}
  - often, from the point of dimensioning, the difference is not important
  - losses depend very strongly on the capacity
  - conversely, the required capacity is not very sensitive to the level of accepted losses

Elementary consideration – normal approximation

- There are n virtual channel connections over the link
  - we assume the streams are statistically identical and independent

$$X = X_1 + \ldots + X_n$$

- Denote

$$\begin{cases} m & = \text{ mean rate of a single stream, } \mathrm{E}[X_1], \\ \sigma^2 & = \text{ variance of the rate of a single stream, } \mathrm{V}[X_1]. \end{cases}$$

- Correspondingly

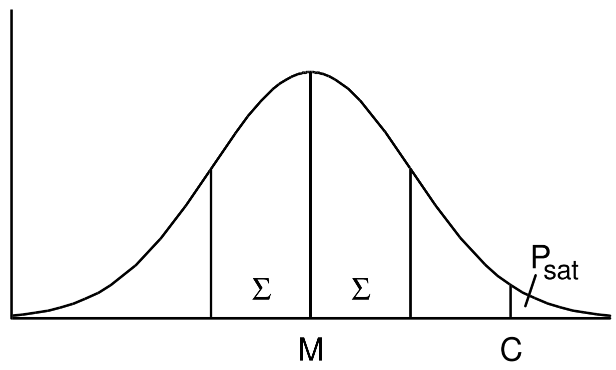
$$\begin{cases} M & = n \cdot m & = \text{mean rate of the aggregate stream, } \mathrm{E}[X], \\ \Sigma^2 & = n \cdot \sigma^2 & = \text{variance of the rate of the aggregate stream, } \mathrm{V}[X] \end{cases}$$

- For large n, approximately $X \sim \mathrm{N}(M, \Sigma^2)$
- Saturation probability is then

$$P_{\text{sat}} = Q\left(\frac{C-M}{\Sigma}\right)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$



Normal approximation – effective bandwidth

- In order not to exceed level P_{sat} requires the capacity C

$$C \geq M + Q^{-1}(P_{\text{sat}}) \cdot \Sigma$$

- Let η be the quantile point of the N(0,1) distribution corresponding to probability $1 - P_{\text{sat}}$

$$\eta = Q^{-1}(P_{\text{sat}})$$

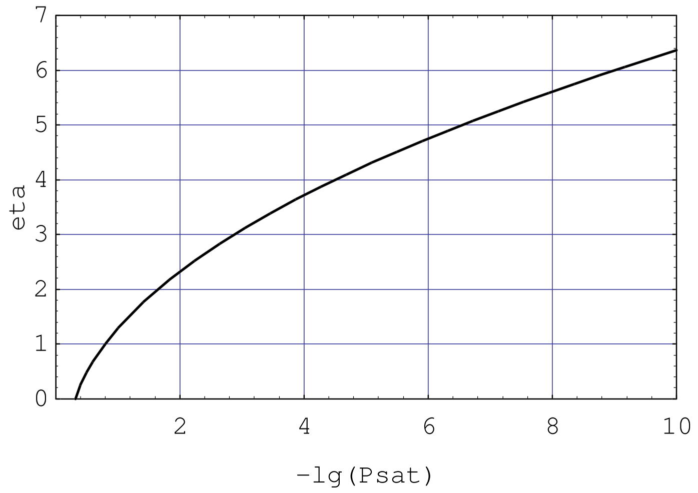
- By the definitions of M and Σ^2 we get

$$C \geq n \cdot m + \eta\sqrt{n} \cdot \sigma$$

- The required bandwidth per stream, so called effective bandwidth, is

$$\boxed{B_{\text{eff}} = m + \frac{\eta \cdot \sigma}{\sqrt{n}}}$$

- As n grows the effective bandwidth tends to the mean rate



Moment generating function

- The moment generating function of a random variable X

  $$M(\beta) = \mathrm{E}[e^{\beta X}] = \int e^{\beta x} f(x) dx$$

- Logarithmic moment generating function

  $$\varphi(\beta) = \log M(\beta)$$

Values at the origin:

$$\begin{cases} M(0) &= 1 \\ \varphi(0) &= 0 \end{cases}$$

- By means of these the moments can be computed as follows

$$\begin{cases} m = \mathrm{E}[X] = M'(0) = \varphi'(0) \\ \mathrm{E}[X^2] = M''(0) \\ \sigma^2 = \mathrm{V}[X^2] = M''(0) - M'(0)^2 = \varphi''(0) \end{cases}$$

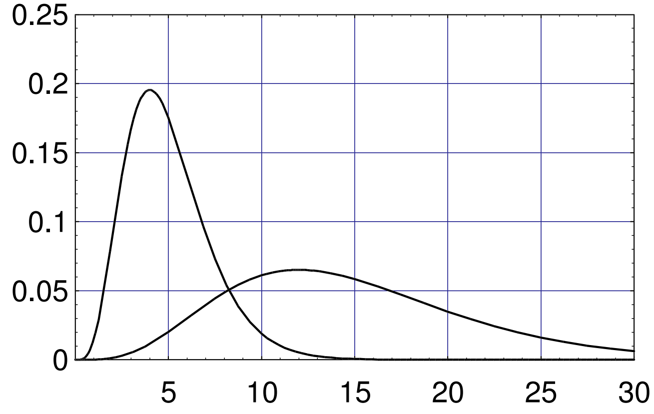
- <u>Additivity</u>: If $X = X_1 + X_2 + \ldots + X_n$, where $X_1,\ X_2, \ldots, X_n$ are independent random variables with lmgfs $\varphi_1(\beta),\ \varphi_2(\beta), \ldots,\ \varphi_n(\beta)$, then

$$\begin{aligned} \varphi(\beta) &= \log \mathrm{E}[e^{\beta X}] \\ &= \log \mathrm{E}[e^{\beta(X_1+X_2+\ldots+X_n)}] \\ &= \log \mathrm{E}[e^{\beta X_1} e^{\beta X_2} \cdots e^{\beta X_n}] \\ &= \log(\mathrm{E}[e^{\beta X_1}] \cdot \mathrm{E}[e^{\beta X_2}] \cdots \mathrm{E}[e^{\beta X_n}]) \\ &= \log \mathrm{E}[e^{\beta X_1}] + \log \mathrm{E}[e^{\beta X_2}] + \ldots + \log \mathrm{E}[e^{\beta X_n}] \\ &= \varphi_1(\beta) + \varphi_2(\beta) + \ldots + \varphi_n(\beta) \end{aligned}$$

Twisted distribution

- Let the pdf of X be $f(x)$

- The twisted or shifted distribution $f_\beta(x)$



$$f_\beta(x) = \frac{e^{\beta x} f(x)}{M(\beta)} = e^{\beta x - \varphi(\beta)} f(x)$$

  - large values of X become more likely
  - the mass of the distribution is shifted towards large values
  - the moment generating function $M(\beta)$ is the normalization factor

- Conversely, $f(x)$ can be expressed in terms of $f_\beta(x)$

$$f(x) = e^{-\beta x + \varphi(\beta)} f_\beta(x)$$

- Twisting can be defined without assuming a continuous distribution

- Twisting of the probability measure

$$dP_\beta(x) = e^{\beta x - \varphi(\beta)} dP(x)$$

- Denote $\mathrm{E}_\beta[\cdot] =$ expectation with respect to the measure P_β

Twisted distribution – continued

- The moments of the twisted distribution

$$\begin{cases} \mathrm{E}_\beta[X] &= \dfrac{\mathrm{E}[Xe^{\beta X}]}{M(\beta)} &= \dfrac{M'(\beta)}{M(\beta)} = \varphi'(\beta) \\ \mathrm{E}_\beta[X^2] &= \dfrac{\mathrm{E}[X^2 e^{\beta X}]}{M(\beta)} &= \dfrac{M''(\beta)}{M(\beta)} \\ \mathrm{V}_\beta[X] &= &= \dfrac{M''(\beta)}{M(\beta)} - \dfrac{M'(\beta)^2}{M(\beta)^2} \;=\; \varphi''(\beta) \end{cases}$$

$$\begin{cases} m(\beta) &= \varphi'(\beta) \\ \sigma^2(\beta) &= \varphi''(\beta) \end{cases}$$

Twisted distribution – continued

- Additivity of the twisted distributions: If $X = X_1+X_2+\ldots+X_n$, where $X_1,\ X_2,\ldots,X_n$ are independent, then it follows from the additivity of the lmg functions

$$\begin{cases} m(\beta) &= m_1(\beta) + m_2(\beta) + \ldots + m_n(\beta) \\ \sigma_1^2(\beta) &= \sigma_1^2(\beta) + \sigma_2^2(\beta) + \ldots + \sigma_n^2(\beta) \end{cases}$$

  where $m_i(\beta)$ and $\sigma_i^2(\beta)$ stand for the expectation and variance of the random variable X_i with respect to the twisted distribution

- In fact, a stronger relation holds:

  The twisted distribution of the sum X of independent random variables X_i equals the sum of the twisted distributions of the individual variables:

$$\begin{array}{ccc} & \text{sum} & \\ & \Longrightarrow & \\ \text{twisting} \Downarrow & & \Downarrow \text{twisting} \\ & \Longrightarrow & \\ & \text{sum} & \end{array}$$

Twisted distribution – example 1

- Exponential distribution $X \sim \mathrm{Exp}(\lambda)$
- Probability density function $f(x) = \lambda e^{-\lambda x}$

$$\begin{cases} M(\beta) &= \mathrm{E}[e^{\beta X}] = \int_0^\infty \lambda \, e^{-(\lambda-\beta)x} dx = \dfrac{\lambda}{\lambda-\beta}, \\ \varphi(\beta) &= \log M(\beta) = \log\lambda - \log(\lambda-\beta) \end{cases}$$

$$\begin{cases} m(\beta) &= \varphi'(\beta) = \frac{1}{\lambda-\beta} \\ \sigma^2(\beta) &= \varphi''(\beta) = \frac{1}{(\lambda-\beta)^2} \end{cases}$$

- The twisted distribution

$$f_\beta(x) = (\lambda-\beta)e^{-(\lambda-\beta)x}$$

  – is also exponential

  – with parameter $\lambda - \beta$

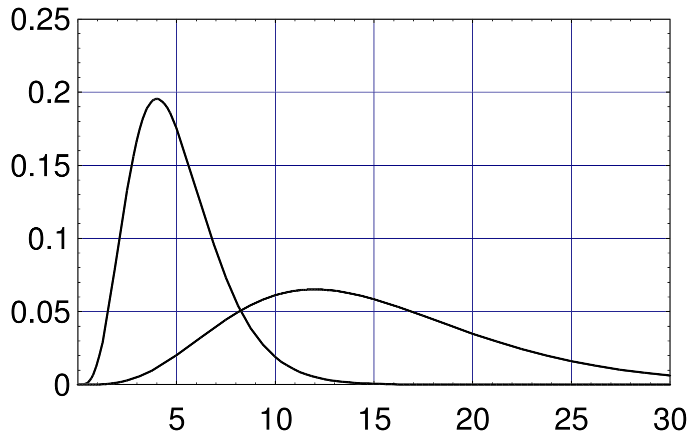
Twisted distribution – example 2

- X is the sum of n independent exponentially distributed random variables
- $X \sim \mathrm{Erlang}(n, \lambda)$
- The density function is

$$f(x) = \lambda \frac{(\lambda x)^{n-1}}{(n-1)!} e^{-(\lambda x)}$$

- Twisted distribution:
  the distribution of the sum of n variables
  with the twisted distributions $\mathrm{Exp}(\lambda - \beta)$
- $\mathrm{Erlang}(n, \lambda - \beta)$ distribution
- Expectation and variance

$$\begin{cases} m(\beta) = \dfrac{n}{\lambda - \beta} \\ \sigma^2(\beta) = \dfrac{n}{(\lambda - \beta)^2} \end{cases}$$



The distribution of the sum of five Exp(1) random variables and the corresponding twisted distribution with the twisting parameter $\beta = \frac{2}{3}$.

Twisted distribution – Summary table

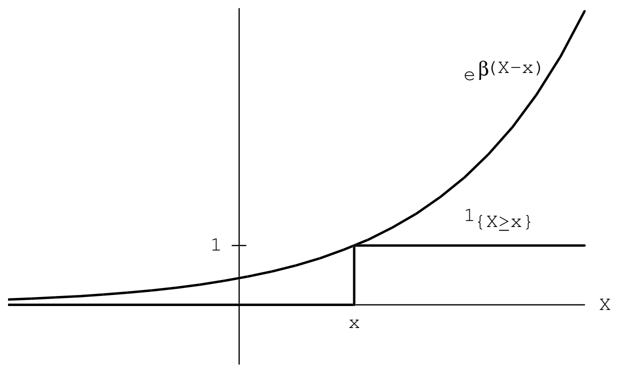
- For the usual families of distributions the twisted distribution belongs to the same family
- The expectation and variance of the twisted distribution are given by the familiar formulae of that family

Distribution	Twisted	$m(\beta)$	$\sigma^2(\beta)$
$\mathrm{Bin}(n,p)$	$\mathrm{Bin}(n, \frac{pe^\beta}{1-p+pe^\beta})$	$\frac{npe^\beta}{1-p+pe^\beta}$	$\frac{n(1-p)pe^\beta}{(1-p+pe^\beta)^2}$
$\mathrm{Erlang}(n,\lambda)$	$\mathrm{Erlang}(n,\lambda-\beta)$	$\frac{n}{\lambda-\beta}$	$\frac{n}{(\lambda-\beta)^2}$
$\mathrm{Poisson}(a)$	$\mathrm{Poisson}(ae^\beta)$	ae^β	ae^β
$\mathrm{N}(m,\sigma^2)$	$\mathrm{N}(m+\sigma^2\beta,\sigma^2)$	$m+\sigma^2\beta$	σ^2

Taulukko 1: The twisted distributions and their parameters for a few distributions.

- Bernoulli distribution is a special case of the binomial distribution: $\mathrm{Bernoulli}(p) \sim \mathrm{Bin}(1,p)$
- Exponential distribution is a special case of the Erlang distribution: $\mathrm{Exp}(\lambda) \sim \mathrm{Erlang}(1,\lambda)$
- χ^2 distribution is a special case of the Erlang distribution: $\chi^2(n) \sim \mathrm{Erlang}(\frac{n}{2},\frac{1}{2})$

The Chernoff bound



- For all $\beta \geq 0$ it holds

$$\begin{aligned} \mathrm{P}\{X \geq x\} &= \mathrm{E}[1_{\{X \geq x\}}] \\ &\leq \mathrm{E}[e^{\beta(X-x)}] \\ &= e^{-\beta x}\mathrm{E}[e^{\beta X}] \\ &= e^{\varphi(\beta)-\beta x} \end{aligned}$$

  since $e^{\beta(X-x)} \geq 1_{\{X \geq x\}}$.

- Then it holds also $\mathrm{P}\{X \geq x\} \leq \inf_\beta e^{-\beta x + \varphi(\beta)}$
- Denote by β_x the value of β which realizes the minimum.
- The Chernoff bound

$$\boxed{\mathrm{P}\{X \geq x\} \leq e^{-\beta_x x + \varphi(\beta_x)}}$$

- The value β_x can be found by minimizing the exponent.

$$\varphi'(\beta_x) = x \quad \text{or} \quad m(\beta_x) = x$$

- The mean of the twisted distribution is shifted to x.

Cramér's theorem

- Let $X = X_1 + X_2 + \ldots + X_n$
  - X_i are independent and identically distributed
  - the component rvs have a common lmgf $\varphi(\beta)$
  - the lmgf of X is $n\,\varphi(\beta)$
- Consider the exceedance probability of the average value $\frac{1}{n}X$

$$\mathrm{P}\{\frac{1}{n}(X_1 + X_2 + \ldots + X_n) \geq x\}$$

- Apply the Chernoff bound

$$\mathrm{P}\{\frac{1}{n}(X_1 + X_2 + \ldots + X_n) \geq x\} = \mathrm{P}\{X \geq n\,x\} \leq e^{-n(\beta_x x - \varphi(\beta_x))}$$

- β_x is determined by the condition $n\,\varphi'(\beta_x) = n\,x,$ that is $\quad \varphi'(\beta_x) = x.$
- The rate function $I(x) = \sup_\beta(\beta x - \varphi(\beta)) = \beta_x x - \varphi(\beta_x)$.
- The exceedance probability of the average

$$\mathrm{P}\{\frac{1}{n}(X_1 + X_2 + \ldots + X_n) \geq x\} \leq e^{-nI(x)}$$

- Cramérs theorem: the upper bound is asymptotically exact in the sense that

$$\boxed{\lim_{n\to\infty} \frac{1}{n} \log \mathrm{P}\{\frac{1}{n}(X_1 + X_2 + \ldots + X_n) \geq x\} = -I(x)}$$

Summary table

- The table gives
  - twisting parameter corresponding to level x
  - the variance of the twisted distribution with parameter β_x
  - rate function $I(x)$

Distribution	β_x	$\sigma^2(\beta_x)$	$I(x)$
$\mathrm{Bin}(n,p)$	$\log \frac{\frac{x}{n}(1-p)}{(1-\frac{x}{n})p}$	$x(1-\frac{x}{n})$	$x \log \frac{\frac{x}{n}(1-p)}{(1-\frac{x}{n})p} + n \log \frac{1-\frac{x}{n}}{1-p}$
$\mathrm{Erlang}(n,\lambda)$	$\lambda - \frac{n}{x}$	$\frac{x^2}{n}$	$x\lambda - n - n\log(\frac{x\lambda}{n})$
$\mathrm{Poisson}(a)$	$\log \frac{x}{a}$	x	$x \log \frac{x}{a} + a - x$
$\mathrm{N}(m,\sigma^2)$	$\frac{x-m}{\sigma^2}$	σ^2	$\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2$

Improved approximation

- The Chernoff bound in terms of the twisted distribution

$$\begin{aligned} \mathrm{P}\{X \geq x\} &= \int_x^\infty f(y)dy \\ &= \int_x^\infty e^{-\beta y+\varphi(\beta)} f_\beta(y)dy \\ &= e^{-\beta x+\varphi(\beta)} \int_x^\infty e^{-\beta(y-x)} f_\beta(y)dy \end{aligned}$$

- In the domain of integration holds $e^{-\beta(y-x)} \leq 1$
  $\Rightarrow$ the whole integral ≤ 1
  $\Rightarrow \mathrm{P}\{X \geq x\} \leq e^{-\beta x+\varphi(\beta)}$
  $\Rightarrow$ The Chernoff bound $e^{-\beta_x x+\varphi(\beta_x)}$ is obtained by minimizing with respect to β

- The tightest bound is obtained at β_x, where $m(\beta_x) = x$
  - the mean of distribution $f_{\beta_x}(\cdot)$ is at x

- Close to the mean the normal distribution $\mathrm{N}(x, \sigma^2(\beta_x))$ is a reasonable approximation

$$f_{\beta_x}(y) \approx \frac{e^{-\frac{1}{2}(y-x)^2/\sigma^2(\beta_x)}}{\sqrt{2\pi}\sigma(\beta_x)}$$

Improved approximation (continued)

- Then (assuming $\beta_x \sigma(\beta_x) \gg 1$)

$$\begin{aligned}\int_x^\infty e^{-\beta_x(y-x)} f_{\beta_x}(y)dy \;&\approx\; \frac{1}{\sqrt{2\pi}\sigma(\beta_x)} \int_x^\infty e^{-\beta_x(y-x)} e^{-\frac{1}{2}(y-x)^2/\sigma^2(\beta_x)} dy \\ &\approx\; \frac{1}{\sqrt{2\pi}\beta_x\sigma(\beta_x)}\end{aligned}$$

- We obtain a (rather accurate) approximation – no longer a strict upper bound

$$\boxed{\mathrm{P}\{X \geq x\} \approx \frac{e^{-I(x)}}{\sqrt{2\pi}\beta_x\sigma(\beta_x)}}$$

The exceedance probability and the loss probability

- According to the more accurate approximation we have for the mean

$$\mathrm{P}\{\frac{1}{n}(X_1 + X_2 + \ldots + X_n) \geq x\} \approx \frac{e^{-nI(x)}}{\sqrt{2\pi n}\,\beta_x \sigma(\beta_x)}$$

  - $I(x)$ is the rate function of one component variable
  - $\sigma(\beta_x)$ is the twisted variance of one component variable
  - the variance of the twisted distribution of the sum is $\sqrt{n}$ times larger
  - therefore we have a facto n in the denominator

- P_{loss} can be approximated as follows

$$P_{\text{loss}} = \frac{1}{m}\mathrm{E}[(X-x)^+] = \frac{1}{m} e^{-\beta x + \varphi(\beta)} \int_x^\infty (y-x) e^{-\beta(y-x)} f_\beta(y)dy$$

- Specifically, set β equal to β_x

$$\int_x^\infty (y-x)e^{-\beta(y-x)} f_\beta(y)dy \approx \frac{1}{\sqrt{2\pi}\sigma(\beta_x)} \int_x^\infty (y-x) e^{-\beta_x(y-x)} e^{-\frac{1}{2}(y-x)^2/\sigma^2(\beta_x)} dy \approx \frac{1}{\sqrt{2\pi}\beta_x^2 \sigma(\beta_x)}$$

  from which it follows

$$\boxed{P_{\text{loss}} = \frac{1}{m}\mathrm{E}[(X-x)^+] \approx \frac{e^{-I(x)}}{\sqrt{2\pi} m \beta_x^2 \sigma(\beta_x)}}$$

Example: calculating the exceedance probability

- What is the probability that the sum of five rvs with distribution Exp(1) exceeds $x = 15$?
- $X \sim \text{Erlang}(5, 1)$
- Exact result: $\text{P}\{X \geq 15\} = 22403e^{-15}/8 \approx 8.566\,10^{-4}$
- We know that $\text{E}[X] = \text{V}[X] = 5$
- <u>Normal approximation</u>, $X \sim N(5, 5)$: $P_{\text{sat}} \approx Q(\frac{15-5}{\sqrt{5}}) \approx 3.87\,10^{-6}$
- <u>The Chernoff bound</u>: $P_{\text{sat}} \leq e^{-5I(15/5)} \approx 1.10\,10^{-2}$

  where $I(x) = x - 1 - \log x$ is the rate function of one component
- <u>Improved approximation</u>: $$P_{\text{sat}} \approx \frac{e^{-5I(15/5)}}{\sqrt{2\pi 5}(2/3)(15/5)} \approx 9.84\,10^{-4}$$

where $\begin{cases} I(x) = x - 1 - \log x \text{ is the rate function of a single component} \\ \beta_x = 1 - 1/3 = 2/3 \\ \sigma^2(\beta_x) = x^2 \end{cases}$

  - the normal approximation is too optimistic
  - the Chernoff bound is very conservative
  - the improved approximation is reasonably accurate

Call acceptance – the effective bandwidth

$$\begin{cases} K & = \text{number of source types; the index of type } k = 1, \dots, K \\ \varphi_k(\beta) & = \text{the logarithmic moment generating function of source type } k \text{ (assumed to be known)} \\ n_k & = \text{the number of sources of type } k \\ c & = \text{the capacity of the link} \end{cases}$$

Acceptance condition according to the loss criterion: $P_{\text{loss}} \leq \epsilon$ (ϵ is the largest allowed value)

Loss probability can be estimated as presented above

$$P_{\text{loss}} \approx \frac{e^{-I(c)}}{\sqrt{2\pi}\, m\, \beta^2\, \sigma(\beta)}$$

where

$$\begin{cases} I(c) & = \beta\, c - \sum_k n_k\, \varphi_k(\beta) \\ \sigma^2(\beta) & = \sum_k n_k\, \sigma_k^2(\beta) = \sum_k n_k\, \varphi_k''(\beta) \end{cases}$$

and β is determined by the condition

$$m(\beta) = c \qquad \text{or} \qquad \sum_k n_k\, \varphi_k'(\beta) = c$$

In order to estimate the loss probability one only needs to solve this equation (numerically) and insert the β in the previous expressions. – This very easy as compared to the convolution of distributions ($\sum_k n_k - 1$ convolutions) as needed in an exact calculation.

Call acceptance – effective bandwidth (continued)

The vector $\mathbf{n} = (n_1, \ldots, n_K)$ composed of the number of connections of each type defines the traffic mix (traffic profile).

For a given mix $\mathbf{n}$ and capacity c we can calculate P_{loss},

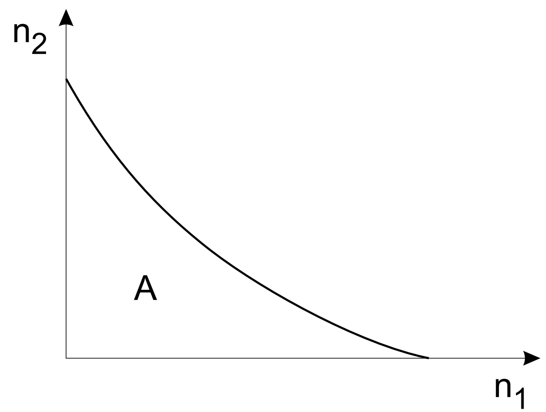
$$P_{\text{loss}} = P_{\text{loss}}(\mathbf{n}, c)$$

By means of this we can conversely find the allowed region A (in general concave)

$$A = \{\mathbf{n} : \; P_{\text{loss}}(\mathbf{n}, c) \leq \epsilon\}$$

that is, all allowed traffic mixes for which $P_{\text{loss}} \leq \epsilon$.

The system can accept new connections as long as $\mathbf{n}$ is in A.



Call acceptance – effective bandwidth (continued)

If the traffic streams do not differ too much, the boundary of the allowed region is approximately linear (a hyperplane)

$$\sum_k n_k B_k \le c, \quad \text{where the effective bandwidth } B_k \text{ is } \quad B_k = \frac{c}{n_k(c)}$$

and $n_k(c)$ tells how many connections of type k alone can be accepted to the link c such that $P_{\text{loss}} \le \epsilon$.

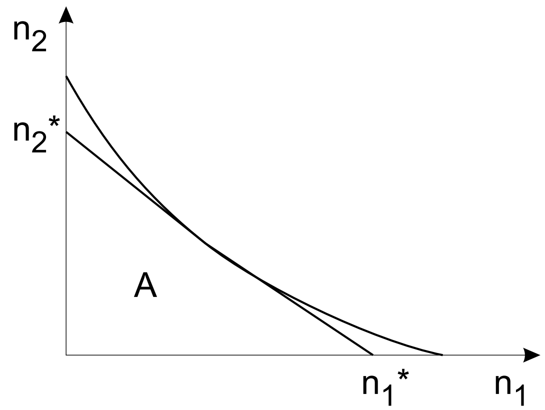
If the traffic types are very different, then the planar approximation is no longer accurate.

An arbitrary tangent plane of the allowed region defines a safe region of acceptance.

We can take as the effective bandwidth the value

$$B_k = c/n_k^*$$

where n_k^* is the point where the tangent plane intersects the n_k axis.



A rough approximation for the effective bandwidth

In order of magnitude calculations one can use Lindberger's approximate formula

$$\boxed{B_k = 1.2m_k + 60\sigma_k^2/c}$$

If only the mean rate m_k and the peak rate h_k are known, we can further make the worst case analysis:

- Suppose the source is of the "on/off" type
  - when the source is on, it sends with the peak rate h_k
  - the source is intermittently off, such that the mean rate is m_k

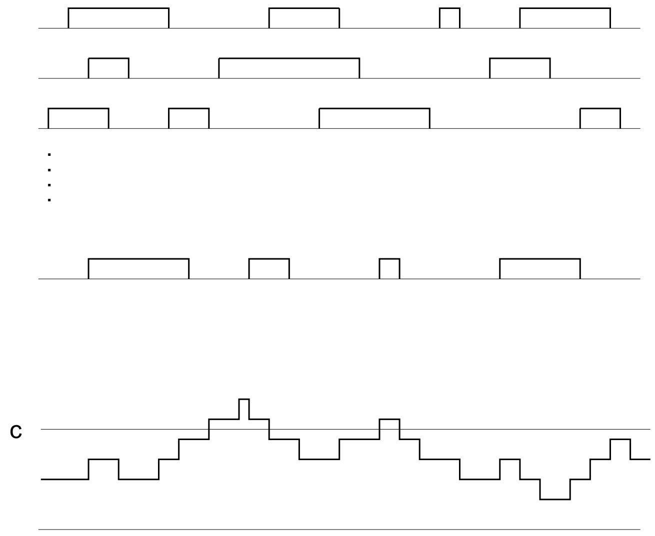
Then one can easily show that

$$\boxed{\sigma_k^2 = m_k(h_k - m_k)}$$

Effective bandwidth in case of on/off sources

The method based on the large deviation theory gives a rather accurate way to estimate the overflow probability for an arbitrary traffic mix.

In some cases the overflow probability can be calculated exactly. The most important case is that of similar "on/off" sources.



- n statistically identical sources
- the rate of a source in the "on" state is h (peak rate)
- the probability of the "on" state is α
- the probability of the "off" state is $1 - \alpha$
- the mean rate of a source is $m = \alpha\, h$

Using the peak rate allocation, one can admit n_0 sources: $n_0 = c/h$

When the number of sources is large and α is not too close to 1, it is very unlikely that all the sources would be simultaneously in the " on" state.

- The sources are statistically multiplexed (interleaved).
- By allowing overflow with a small probability, one can significantly increase the allowed number of connections (so called multiplexing gain G).

Effective bandwidth in case of on/off sources (continued)

Probability p_i that i sources out of n are simultaneously active

$$p_i = \binom{n}{i} \alpha^i (1-\alpha)^{n-i}$$

The overflow probability is thus

$$P_{\text{loss}}(n, n_0, \alpha) = \frac{1}{nm} \sum_{i=\lceil n_0 \rceil}^{n} p_i \cdot (i - n_0) h = \frac{1}{n\alpha} \sum_{i=\lceil n_0 \rceil}^{n} p_i \cdot (i - n_0)$$

Largest allowable number of connections $n_\epsilon(n_0, \alpha)$ is determined from the condition

$$P_{\text{loss}}(n_\epsilon, n_0, \alpha) \leq \epsilon$$

The effective bandwidth B_{eff} of a single connection is $(m \leq B_{\text{eff}} \leq h)$

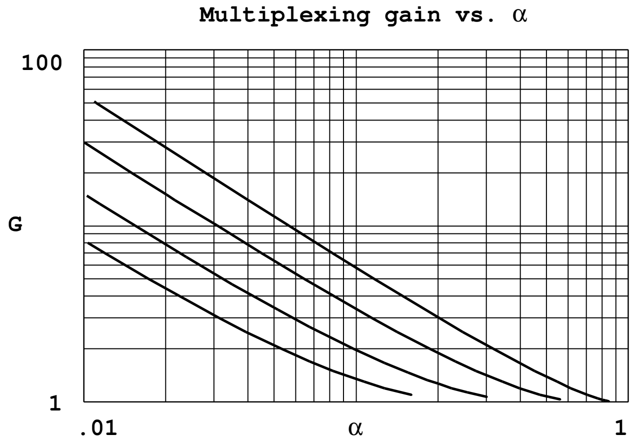
$$B_{\text{eff}} = \frac{c}{n_\epsilon} = \frac{n_0}{n_\epsilon} \cdot h$$

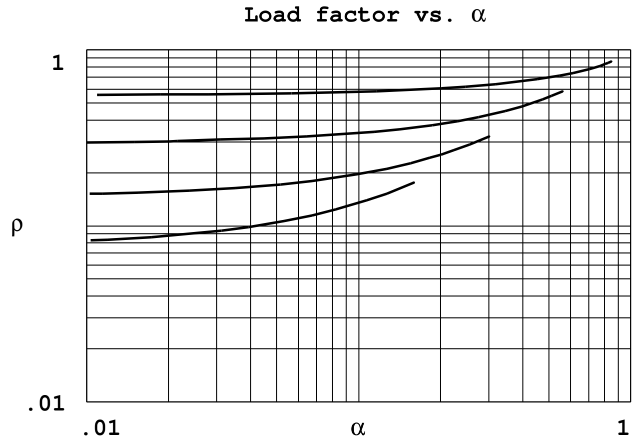
- Multiplexing gain $G = n_\epsilon / n_0 = h / B_{\text{eff}}$
- Allowed load $\rho = n_\epsilon m / c = n_\epsilon \alpha h / c = \alpha n_\epsilon / n_0 = \alpha G$

Effective bandwidth in case of on/off sources (continued)

The effectiveness of the statistical multiplexing can be gauged by means of either the multiplexing gain G or the allowed load $\rho = \alpha G$ (both contain the same information).

- The figures below show G and ρ as a function of α with $\epsilon = 10^{-9}$.
- The parameter of the family of curves is $n_0 = c/h$ (from below 10, 15,30,100).





- For small α (bursty traffic) the multiplexing gain can be great.
- The allowed load, however, depends almost solely on the parameter c/h.
- In order to have a reasonable load in a bufferless system it is required that the peak rate of every single source is a small fraction of the link rate (1% or less).