HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, Fall 2005

Exercise 4
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1. Million (10^6) data packets per second arrive at a network from different sources. The lengths of routes, defined by the source and destination addresses, vary considerably. The time a packet spends in the network depends on the length of the route, but also on the congestion of the network. The distribution of the time a packet spends in the network is assumed to have the following distribution: 1 ms (90 %), 10 ms (7 %), 100 ms (3 %). How many packets there are in the network on average?

2. An Erlang loss system consisting of five trunks ($M/M/5/5$-system) receives an arriving traffic stream with intensity $a = 5$ Erl. The blocked calls are offered to be carried on an overflow trunk group. a) What is the intensity of the overflow traffic? b) Which fraction of the overflow traffic is blocked in the overflow trunk group? Hint: The original trunk group and the overflow trunk group together form a $M/M/6/6$-system. c) What would the blocking probability be on the overflow trunk group if the overflow traffic offered to it were Poissonian (whence the blocking can be calculated with Erlang formula). Explain the difference.

3. Ordered search in an n server Erlang system: Assume that the servers are labeled sequentially with numbers $1, \ldots, n$ nd the offered load is a erl. Each arriving customers goes to the free server with the lowest number. What proportion of the time is the server i in use? Hint: Note, that all the servers $1, \ldots, i$ are in use with probability of $E(i, a)$, and deduce from this the average arrival rate to servers $i + 1, \ldots, n$, and finally the arrival rate to each server.

4. Consider 2×1- and 4×2-concentrators, where for each input port calls arrive according to independent Poisson-processes with intensities γ. The mean call holding time is denoted by $1/\mu$ and the offered load by $\hat{a} = \gamma/\mu = 0.1$. Compare in these two concentrators the probabilities that a call arriving to a free input port gets blocked because all the output ports are busy.

5. Assume that there is always one doctor at a clinic (24 hours a day) and that customers arrive according to a Poisson process with intensity λ. The service time of each customer obeys an exponential distribution with mean 30 minutes. Determine the maximum arrival intensity that allows 95% of the customers to be served within 72 hours from (the first) arrival.

6. Consider an M/M/1/K queue with states $0, 1, \ldots, K$. Find the probability P_n that the queue which initially is in the state n becomes empty before flowing over. Hint: Add an imaginary state $K + 1$ to the system; a transition to the state $K + 1$ corresponds to the queue flowing over. We have $P_0 = 1$ and $P_{K+1} = 0$. For states $n = 1, \ldots, K$, write the probability P_n in terms of P_{n-1} ja P_{n+1}. Solve the equations.