HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 5**
Networking Laboratory | 6.12.2005
S-38.3143 Queueing Theory, Fall 2005 | Virtamo / Koskinen

1. A mail-order company receives calls at a Poisson rate of one per 2 min and the duration of the calls is exponentially distributed with mean 3 min. A caller who finds all telephone operators busy patiently waits until one becomes available. How many operators the company should use so that the average waiting time of a customer is half a minute or less?

2. If in a single server system each customer has to pay a fee to the system according to some rule, then the average revenue rate of the system = $\lambda \cdot (\text{average fee})$, where λ is the mean rate of arriving customers.

   Apply this to the $M/G/1$ system with the following charging rule: each customer in the system pays at rate which is the same as the customer's remaining service time. What is the average fee? Show by equating the above average revenue rate with the average charging rate (time charging) that
   $$\overline{W} = \lambda\,(\overline{X}\,\overline{W} + \overline{X^2}/2),$$
   where W and X are the waiting and service times. Solve $\overline{W}$. What is this result?

3. Customers arrive at an $M/\mathrm{Erlang}(k,\mu)/1$ system according to a Poisson process with rate λ. Find the mean waiting and sojourn times of a customer in the system?

4. Persons arrive at a copying machine according to a Poisson process with rate 2/min. The number of copies to be made by each person is uniformly distributed between $1, \ldots, 5$. Each copy takes 4 s. Find the average waiting time in the queue when

   a) Each person uses the machine on a first-come first-served basis.
   b) Persons with no more than 1 copy to make are given non-preemptive priority over other persons.

5. Consider a priority queue with two classes and preemptive resume priority. Customers arrive according to two independent Poisson processes with intensities λ_1 and λ_2. Service times in both classes are independent and exponentially distributed with a joint mean $1/\mu$. Determine the mean sojourn times $\bar{T}_1$ and $\bar{T}_2$ for both classes.

6. Consider a n-class, non-preemptive priority system: Suppose there is a cost c_k per unit time for each class k customer that waits in queue. Show that cost is minimized when classes are ordered so that
   $$\frac{\overline{S}_1}{c_1} \leq \frac{\overline{S}_2}{c_2} \leq \ldots \leq \frac{\overline{S}_n}{c_n},$$
   where $\overline{S}_k$ is the average service time of class-k customer.

   Hint: Express the cost as $\sum_k \left(\frac{c_k}{\overline{S}_k}\right)\left(\rho_k \overline{W}_k\right)$ and apply Kleinrock's conservation law for M/G/1. Also use the fact that interchanging the order of any two adjacent classes leaves the waiting time of all other classes unchanged.