HELSINKI UNIVERSITY OF TECHNOLOGY — **Exercise 2**
Networking Laboratory — 14.11.2006
S-38.3143 Queueing Theory, II/2006 — Virtamo / Penttinen

1. When a ticket sales office opens, some customers are already queuing. The number of customers N already in the queue has the distribution $\mathrm{P}\{N = i\} = (1-\rho)\rho^{i-1}$, $i = 1, \ldots$. The number of new customers arriving at the queue during a customer service time has the distribution $\mathrm{P}\{X = i\} = (1-p)p^i$, $i = 0, \ldots$. Let Y denote the number of new customers who arrive during the time it takes to serve all the the original customers, i.e. the number of customers present at the moment when the last customer who was there already before opening has been served. Determine the distribution of Y. Note the difference in the definitions of the distributions, $N \geq 1$ and $X \geq 0$.

2. Let $X_1, \ldots, X_n$ $(n \geq 2)$ be a set of independent Exp(λ)-distributed random variables. Let random variable R denote,

$$R = \max(X_1, \ldots, X_n) - \min(X_1, \ldots, X_n).$$

   Deduce that cumulative distribution function of R is $\mathrm{P}\{R \leq x\} = (1 - e^{-\lambda x})^{n-1}$.

3. Calls arrive to a modem pool with exponentially distributed time intervals with mean 15 s. What is the probability that starting from an arbitary instant of time it takes more than 30 seconds until the 3^{rd} call arrives?

4. When customer A arrives at a bank, all the four service points are reserved but there are no other customers in the queue. The customer service times are assumed to be independent and exponentially distributed with the mean 1 min.

   a) Assume that no new customers arrive. What is the probability that A is the last customer leaving the bank? Would the result change if there is a queue before A and the customers are served in their arrival order?
   b) What is the expected value of the time A spends in the bank?
   c) On average, what is the time from the arrival of A to the moment when the last customer leaves, if nobody arrives after A?

5. A Markov chain with states $1, \ldots, 4$ has the following transition probability matrix:

$$\mathbf{P} = \begin{pmatrix} 1-p & 0 & p & 0 \\ q & 0 & q & 0 \\ 0 & 0 & 1-p & p \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

   a) What must the value of q be?
   b) Draw the state transition diagram of the system and classify the states.
   c) What is the probability that the system is in state 4 at time 4 assuming it is in state 2 at time 2?

6. Consider again the first problem of the previous exercise. A connection consists of 4 unreliable consecutive links. On each link the probability that a transmitted bit (0 or 1) is received correctly is 90% and with probability of 10% the received bit has flipped into the other one. Model the state of a bit with a Markov chain and utilize the model to determine the probability that a transmitted bit is received correctly at the other end of the connection?