HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, II/2006

Exercise 4
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1. Consider a car-inspection center where cars arrive at a rate of one every 50 seconds and wait for an average of 15 minutes (inclusive of inspection time) to receive their inspections. After an inspection, 20 % of the car owners stay back an average of 10 minutes having their meals at the center's cafeteria. What is the average number of cars within the premises of the inspection center (inclusive of the cafeteria)?

2. Consider a communications link with unbounded capacity. At $t = 0$ the link is empty and calls start arriving according to Poisson process with the intensity λ. The call holding times are assumed to be independent and exponentially distributed with the mean $1/\mu$. Consider the number of on-going calls N_t at the time instant $t \geq 0$. Determine the distribution of N_t and, in particular, its mean as a function of time t. Hint: The number of on-going calls equals the number of arrivals between $(0, t)$ from a certain inhomogeneous Poisson process, which can be obtained from the original Poisson process with a suitable random selection.

3. An Erlang loss system consisting of five trunks ($M/M/5/5$-system) receives an arriving traffic stream with intensity $a = 5$ Erl. The blocked calls are offered to be carried on an overflow trunk group. a) What is the intensity of the overflow traffic? b) Which fraction of the overflow traffic is blocked in the overflow trunk group? Hint: The original trunk group and the overflow trunk group together form a $M/M/6/6$-system. c) What would the blocking probability be on the overflow trunk group if the overflow traffic offered to it were Poissonian (whence the blocking can be calculated with Erlang formula). Explain the difference.

4. A statistician is observing an Erlang loss system, which has s servers and the offered load is a erl. The observation time starts at a random time instant and the observation is continued until the next customer arrives at the system. What is the probability that this customer is blocked? Why isn't the probability equal to $E(s, a)$?

5. Customers are arriving at an $M/M/n/n$ system with the rate λ. The offered load is a.

   a) What is the mean time interval between loss events?
   b) Assume that the system is at state $i < n$. How long it takes on average before the system enters the state $i + 1$ for the first time? Hint: Examine a system with only i servers.
   c) What is the mean lenght of a blocking interval (the system is continuously in a blocking state)?
   d) How many customers are blocked, on average, during a blocking interval?
   e) What is the mean (non-blocking) interval between two blocking intervals?

6. Ten telephone subscribers are connected to the input side of a concentrator, each by his own subscriber line. During the busy hour each subscriber generates calls with mean duration 3 min and the interval from the termination of a call to a new call attempt is exponentially distributed with mean 30 min. How many lines are needed on the output side of the concentrator, when it is required that the blocking probability of a call attempt, due to insufficient number of lines, shall be less than or equal to 0.01?