HELSINKI UNIVERSITY OF TECHNOLOGY **Exercise 6**
Networking Laboratory 12.12.2006
S-38.3143 Queueing Theory, II/2006 Virtamo / Penttinen

1. Starting from a mean waiting time formula for non-preemptive priority queues (cf. lecture notes) show by direct calculations that quantity $\sum_{k=1}^{K} \rho_k \overline{W}_k$ is a constant and does not depend on how the classes are set.

   Hint: Let $R_k = \sum_{i=1}^{k} \rho_i$ and determine first $\frac{1}{1-R_k} - \frac{1}{1-R_{k-1}}$.

2. Consider a $n$-class, non-preemptive priority system: Suppose there is a cost $c_k$ per unit time for each class $k$ customer that waits in queue. Show that cost is minimized when classes are ordered so that

$$\frac{\overline{S}_1}{c_1} \leq \frac{\overline{S}_2}{c_2} \leq \cdots \leq \frac{\overline{S}_n}{c_n},$$

   where $\overline{S}_k$ is the average service time of class-$k$ customer.

   *Hint*: Express the cost as $\sum_k \left(\frac{c_k}{\overline{S}_k}\right) \left(\rho_k \overline{W}_k\right)$ and apply Kleinrock's conservation law for M/G/1. Also use the fact that interchanging the order of any two adjacent classes leaves the waiting time of all other classes unchanged.

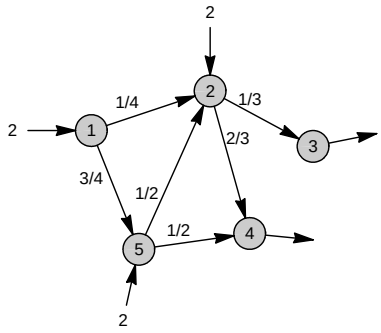
3. The Pollaczek-Khinchin formula for the Laplace transform of the waiting time $W$ is

$$W^*(s) = \frac{s(1-\rho)}{s - \lambda + \lambda S^*(s)}$$

   where $S^*(s)$ is the Laplace transform of the service time $S$ and $\rho = \lambda \overline{S}$. Using this result, rederive the PK mean formula for the waiting time.

4. Apply the Pollaczek-Khinchin transform formula of the previous problem to the $M/D/1$ system, where the service time is constant $d$. Calculate the expectation and variance of the waiting time. Hint: Determine $S^*(s)$, develop it into power series, take an appropriate number of terms and make the division.

5. Consiner the Jackson queueing network depicted below. Packets from outside arrive to the nodes 1, 2 and 5 as a Poisson stream with rate $\lambda = 2$ packets/s. In every node each link has own buffer. The incoming packet stream to each node is randomly directed with the depicted probabilities. The link from node 4 has capacity of $\mu = 8$ packets/s, while the capacity of the other links are $\mu = 3$ packets/s. a) What are the mean delays of packets taking the routes 1-2-3 and 1-5-4? b) How many packets there is on average in the network? c) What is the mean sojourn time of packets entering the network?




6. A closed queueing network consists of three queues in a ring. The service rates of the queues are $\mu$, $2\mu$ and $4\mu$. There are two customers circulating in the ring. Find the mean queue lengths of the queues and the mean round trip time of a customer.