HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 4**
Networking Laboratory | 27.11.2007
S-38.3143 Queueing Theory, II/2007 | Virtamo / Penttinen

1. Consider a car-inspection center where cars arrive at a rate of one every 50 seconds and wait for an average of 15 minutes (inclusive of inspection time) to receive their inspections. After an inspection, 20 % of the car owners stay back an average of 10 minutes having their meals at the center's cafeteria. What is the average number of cars within the premises of the inspection center (inclusive of the cafeteria)?

2. Consider a communications link with unbounded capacity. At $t = 0$ the link is empty and calls start arriving according to Poisson process with the intensity λ. The call holding times are assumed to be independent and exponentially distributed with the mean $1/\mu$. Consider the number of on-going calls N_t at the time instant $t \geq 0$. Determine the distribution of N_t and, in particular, its mean as a function of time t. Hint: The number of on-going calls equals the number of arrivals between $(0, t)$ from a certain inhomogeneous Poisson process, which can be obtained from the original Poisson process with a suitable random selection.

3. A mail order company has 3 persons serving incoming calls. The calls arrive according to a Poisson process with intensity of 1/min and the average call duration is 2 min.

   a) What is the probability that an incoming call is blocked, when blocked calls are lost?

   b) Is it profitable to hire a 4^{th} person if the total expenses per person are 100 €/h and the average revenue per order is 20 €?

4. Use recursive Erlang's blocking formula to calculate $E(n, 6)$ for $n = 0, \ldots, 6$.

5. Consider Erlang's loss system with n servers, where the intensity of the offered traffic is a.

   a) Show by direct calculations based on the steady state probabilities, that the mean number of customers in the system $\overline{N}$ is equal to $(1-\mathrm{E}(n, a))a$. Explain this by using Little's theorem.

   b) By using Little's theorem, deduce the mean time the system stays at state $N = n$ at a time. (Hint: How often the system moves to state $N = n$? The mean number of customers in a given state is the same as steady state probability of the same state; the system either is in that particular state or not.) Deduce the same result directly from the properties of the exponential distribution.

6. Consider 2×1- and 4×2-concentrators, where for each input port calls arrive according to independent Poisson-processes with intensities γ. The mean call holding time is denoted by $1/\mu$ and the offered load by $\hat{a} = \gamma/\mu = 0.1$. Compare in these two concentrators the probabilities that a call arriving to a free input port gets blocked because all the output ports are busy.