HELSINKI UNIVERSITY OF TECHNOLOGY — **Exercise 5 (extra)**
Networking Laboratory — 23.10.2006
S-38.3148 Simulation of data networks, fall 07 — Lassila / Tirronen

Return your answers by 14:00 on Tuesday, Oct 23, in the cupboard labeled S-38.3148 outside the Networking laboratory in the G-wing (2nd floor).

1. Let Y_1 and Y_2 denote independent identically exponentially distributed rv:s with mean $1/\mu$, and let $X = \min(Y_1, Y_2)$. Devise an algorithm with which one can generate samples of X where only one pseudo random number U (with uniform distribution in the range $[0, 1]$) needs to be generated. Hint: the tail distribution of the minimum of a set of rv:s equals the product of the tail distributions of the individual rv:s.

2. a) Let X be a random variable that follows a pareto distribution with shape parameter a, $X \sim \text{Par}(a)$. The density function of X is then

$$f(x) = \frac{a}{x^{a+1}}, \quad x \geq 1$$

It can be shown that a random variable $Z = bX$ also follows a pareto distribution with parameters a and b, where b is a scale parameter. What is the density function of Z? How would you generate random samples of Z?

3. Let the density function of X be

$$f(x) = \begin{cases} \frac{1}{2} & 0 < x < 1 \\ \frac{3}{4} - \frac{x}{4} & 1 \leq x \leq 3 \end{cases}$$

Give an algorithm to generate samples of X by using the inverse transformation technique.

4. Give an algorithm to generate samples of X from the above distribution by using the rejection method.

5. What is the pdf of the samples generated by the following algorithm?

   0. Draw U_1 and U_2 from the distribution $U(0, 1)$
   1. **if** $(U_1 \leq \frac{2}{3})$ **return** $X = \sqrt{U_2}$
   2. **else return** $X = -\frac{1}{2}\ln(U_2)$

6. A bank opens its doors at 9 am and closes its doors at 4 pm. Customers arrive according to Poisson process at rate 1 per minute. All customers arriving during the opening hours will be served. In 10 independent replications of a simulation of a day, the observed daily average waiting times are (2.50, 2.66, 2.24, 3.34, 3.17, 2.69, 3.69, 3.86, 2.70, 3.60) min. Give the point estimate for the daily average waiting time and its 95 % confidence interval?