HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3148 Simulation of data networks, fall 07

Exercise 4
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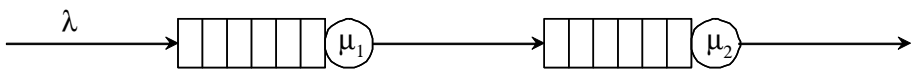
1. Assume that we want to generate uniformly distributed points within the unit circle $x^2 + y^2 = 1$. Give an algorithm based on the inverse transformation technique to directly generate samples of the x- and y-coordinates. Hint: consider x and y in the polar coordinates.

2. Consider a simple tandem queuing network with two queues as shown in the figure below. Jobs arrive according to a Poisson process with rate λ. The job sizes are assumed to obey the log-normal distribution. The queues have service rates μ_1 and μ_2, respectively, and both queues have a finite length of K packets. Hence, the system is a tandem network of M/G/1/K queues. We also assume the system is stable, i.e., $\lambda < \min(\mu_1, \mu_2)$.

   Describe a simulation method based on the regenerative approach for estimating the packet loss probability in this system. How does it affect if the job sizes would obey the exponential distribution? (Note that you are not expected to give a full pseudo code implementation for handling each packet arrival and departure, but just a description how the regenerative method is used in this context.)



Kuva 1: Tandem network with two queues.

3. The random variable Y is defined as follows: $Y = X_1/(1 + X_2/5)$, where $X_1 \sim X_2 \sim$ Exp(1), and X_1 and X_2 are independent. Devise a method based on the use of a control variable for estimating the expectation of Y, E[Y].

   Implement both a direct method and the control variable method. Generate 10000 samples using both methods and compare your point estimates and their 95 % confidence intervals. Please include a listing of your "code"in the answer.

4. Consider the simulation of a GI/GI/1 queue (i.i.d. service and interarrival times with a general distribution). In the simulation of this system one typically uses one sequence of random numbers $U_1, U_2, \ldots$ to generate the arrival times of both classes and the service times. Indicate a good way to simulate the system based on the idea of antithetic variables in order to reduce the variance of the estimates for the waiting times.

5. **Note! This problem is worth 5 p.** Assume that the arrivals occur according to the Poisson process with rate λ and the service times obey the exponential distribution with mean $1/\mu$. The task is to estimate E[N], i.e., the mean number of customers in the system. Implement the direct simulation and the idea of using antithetic variables from problem 4. Simulate the system with load $\rho = 0.9$ (e.g., choose $\mu = 1$ and $\lambda = 0.9$) and

show that your estimate for E[N] has a lower variance when using antithetic variables. Hint: the pseudo code for M/M/1 simulator is in the lecture slides for "Discrete event simulation", slide 8. Include a program listing of your code in the answer.