Helsinki University of Technology
Signal Processing Laboratory

S-38.411 Signal Processing in Telecommunications I

Spring 2000

Lecture 4: Optimal linear equalizers for linear channels 1

Prof. Timo I. Laakso

timo.laakso@hut.fi, Tel. 451 2473

http://wooster.hut.fi/studies.html

Timetable

L1 Introduction; models for channels and communication systems
L2 Channel capacity
L3 Transmit and receive filters for bandlimited AWGN channels
L4 Optimal linear equalizers for linear channels 1
L5 Optimal linear equalizers for linear channels 2
L6 Adaptive equalizers 1
L7 Adaptive equalizers 2
L8 Nonlinear receivers 1: DFE equalizers
L9 Nonlinear receivers 2: Viterbi algorithm
L10 GL1: DSP for Fixed Networks / *Matti Lehtimäki, Nokia Networks*
L11 GL2: DSP for Digital Subscriber Lines / *Janne Väänänen, Tellabs*
L12 GL3: DSP for CDMA Mobile Systems / *Kari Kalliojärvi, NRC*
L13 Course review, questions, feedback
E 24.5. (Wed) 9-12 S4 **Exam**

Contents of Lecture 3

Optimal linear equalizers for linear channels 1

I. Generalized matched filter

II. GMF with Nyquist criterion

III. Nyquist only: Zero-forcing equalization

Helsinki University of Technology
Signal Processing Laboratory

I. Generalized matched filter

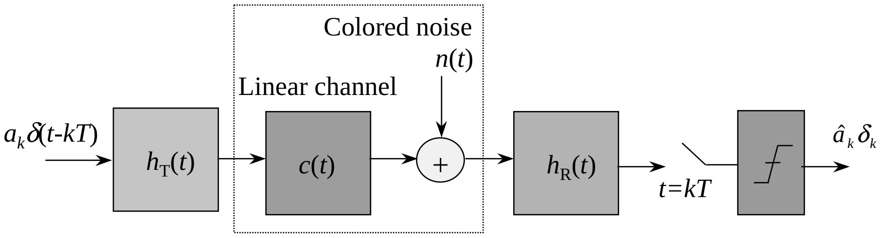
Generalized Matched Filter

- In the previous lecture, we considered transmit and receive filter design for AWGN channels
- Root-Nyquist filters combine the matched filter (max SNR) and the Nyquist criterion (zero ISI) in AWGN channel
- Topic of this lecture:

  How to generalize these ideas for a linear channel with colored noise spectrum?
- Terminology: receiver processing for compensating for linear channel effects is called *equalization*

Generalized Matched Filter...

- System model:



Generalized Matched Filter...

- Notation: $x(t)=\sum_k a_k\delta(t-kT), \quad r(t)=h_T(t)*c(t)*x(t)+n(t)$

$$y(t)=h_R(t)*r(t)=h_R(t)*c(t)*h_T(t)*x(t)+h_R(t)*n(t)$$
$$\equiv g(t)+n_R(t)$$

$x(t)$ = input signal (symbol sequence)

$h_T(t)$, $h_R(t)$ = transmit and receive filters

$c(t)$ = channel impulse response

$n(t)$ = additive Gaussian noise (colored): $S_n(f)=\frac{N_0}{2}S_{n0}(f)$

Normalization: $\int_{-\infty}^{\infty} S_{n0}(f)df=2$

Generalized Matched Filter...

Assumptions:

- Tx filter $h_T(t)$ is fixed
- Rx filter $h_R(t)$ is to be optimized
- Optimization criterion: max SNR at Rx
- Channel $c(t)$ and noise power spectrum $S_n(f)$ known at Rx

Generalized Matched Filter...

- Pulse waveform after receive filter (without noise):

$$g(t) = h_T(t) * c(t) * h_R(t) = \int_{-\infty}^{\infty} H_T(f) C(f) H_R(f) e^{j2\pi ft} df$$

- Pulse energy at sampling instant:

$$g^2(0) = \left| \int_{-\infty}^{\infty} H_T(f) C(f) H_R(f) df \right|^2$$

- Noise power (after Rx filter):

$$E\left[n_R^2(t)\right] = \frac{N_0}{2} \int_{-\infty}^{\infty} \left|H_R(f)\right|^2 S_{n0}(f) df = \frac{N_0}{2} \int_{-\infty}^{\infty} \left|H_R(f)\sqrt{S_{n0}(f)}\right|^2 df$$

Generalized Matched Filter...

- Resulting SNR:

$$SNR = \frac{g^2(0)}{E\left[n_R^2(t)\right]} = \frac{\left| \int_{-\infty}^{\infty} H_T(f) C(f) H_R(f) df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} \left|H_R(f)\sqrt{S_{n0}(f)}\right|^2 df}$$

- Use Schwarz inequality:

$$SNR \leq \frac{\int_{-\infty}^{\infty} \left|H_T(f) C(f) / \sqrt{S_{n0}(f)}\right|^2 df \int_{-\infty}^{\infty} \left|H_R(f)\sqrt{S_{n0}(f)}\right|^2 df}{\frac{N_0}{2} \int_{-\infty}^{\infty} \left|H_R(f)\sqrt{S_{n0}(f)}\right|^2 df}$$

$$= \frac{2}{N_0} \int_{-\infty}^{\infty} \left|H_T(f) C(f) / \sqrt{S_{n0}(f)}\right|^2 df = SNR_{MAX}$$

Generalized Matched Filter...

- Max SNR obtained when:

$$H_{\mathrm{R}}(f)\sqrt{S_{n0}(f)} = k_0\left[\frac{H_{\mathrm{T}}(f)C(f)}{\sqrt{S_{n0}(f)}}\right]^*$$

$$\Leftrightarrow \quad H_{\mathrm{R}}(f) = k_0\,\frac{H_{\mathrm{T}}^*(f)C^*(f)}{S_{n0}(f)}$$

- GMF waveform:

$$h_{\mathrm{R}}(t) = F^{-1}\left\{k_0\,\frac{H_{\mathrm{T}}^*(f)C^*(f)}{S_{n0}(f)}\right\}$$

$$= k_0 h_{\mathrm{T}}(-t) * c(-t) * n_{\mathrm{I}}(t)$$

Generalized Matched Filter...

- GMF waveform interpretation:

$$h_R(t) = k_0 h_{\mathrm{T}}(-t) * c(-t) * n_{\mathrm{I}}(t)$$

k_0 = constant
$h_{\mathrm{T}}(-t)$ = *pulse* matched filter
$c(-t)$ = *channel* matched filter
$n_{\mathrm{I}}(t)$ = noise compensation (NOT whitening!)

Generalized Matched Filter...

- Example of GMF use: RAKE receiver in CDMA
  - chip-level pulse shaping (Tx) & matched filter (Rx)
  - signal spreading with code (Tx) & code-matched filter (Rx)
  - channel (Ch) & channel-matched filter or RAKE (Rx)
- Possible because ISI can be neglected
- Usual problem: ISI! (Not considered by MF at all!)

Helsinki University of Technology
Signal Processing Laboratory

II. GMF with Nyquist Criterion

GMF with Nyquist criterion

Assumptions:

- Tx filter $h_T(t)$ and Rx filter $h_R(t)$ are to be *jointly optimized*
- Optimization criterion: max SNR at Rx with zero ISI constraint (= Nyquist criterion)
- The Nyquist spectrum $G_N(f)$ is chosen (e.g. raised-cosine) and normalized so that peak pulse at Rx $g_N(0) = 1$
- Channel $c(t)$ and noise power spectrum $S_n(f)$ are known both at Tx and Rx

GMF with Nyquist...

- Nyquist constraint:

$$G(f) = G_N(f) = H_T(f)C(f)H_R(f)$$

$$\Leftrightarrow \quad H_T(f) = \frac{G_N(f)}{C(f)H_R(f)} \qquad (1)$$

- GMF solution:

$$H_R(f) = \frac{H_T^*(f)C^*(f)}{S_{n0}(f)} \qquad (2)$$

GMF with Nyquist...

- Combined solution (linear-phase Rx):

$$H_T(f) = \frac{\sqrt{G_N(f)S_{n0}(f)}}{C(f)}, \quad H_R(f) = \sqrt{\frac{G_N(f)}{S_{n0}(f)}}$$

- Pulse spectrum $G(f) = G_N(f)$, $g_N(0) = 1$
- Noise PSD at Rx output: $S_{n,R}(f) = \frac{N_0}{2} G_N(f)$
- Noise power: $E\left[n_R^2(t)\right] = \frac{N_0}{2} \int_{-\infty}^{\infty} G_N(f)\,df = \frac{N_0}{2}$

GMF with Nyquist...

- SNR after Rx:

$$SNR = \frac{g^2(0)}{E\left[n_R^2(t)\right]} = \frac{2}{N_0}$$

- Hence, the SNR is the same as that of an AWGN channel!!!
- The joint design of Tx and Rx filters with GMF&Nyquist thus completely compensates for channel effects!
- Problems:
  - channel needs to be known at Rx&Tx
  - Tx power increase for low-gain channels
  - $C(f) = 0$: Tx filter non-realizable!

Helsinki University of Technology
Signal Processing Laboratory

III. Nyquist only: Zero-forcing equalization

Nyquist only: Zero-forcing equalization

Assumptions:

- Tx filter $h_T(t)$ is fixed
- Rx filter $h_R(t)$ is to be optimized
- Optimization criterion: zero ISI at Rx (Nyquist)
  (→ Noise completely neglected in the design!)
- Channel $c(t)$ and noise power spectrum $S_n(f)$ known at Rx

Nyquist only: ZF equalization...

- Composite pulse spectrum to be Nyquist: $G(f) = G_N(f)$
  - $\rightarrow$ Solution for Rx filter directly:

$$H_R(f) = \frac{G_N(f)}{H_T(f)C(f)}$$

- Assume root-Nyquist Tx filter: $H_T(f) = \sqrt{G_N(f)}$

$$\rightarrow H_R(f) = \frac{\sqrt{G_N(f)}}{C(f)}$$

Nyquist only: ZF equalization...

- Noise power:

$$E\left[n_R^2(t)\right] = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_R(f)\right|^2 S_{n0}(f)df = \frac{N_0}{2}\int_{-\infty}^{\infty}\frac{\left|G_N(f)\right|S_{n0}(f)}{\left|C(f)\right|^2}df$$

- SNR:

$$SNR = \frac{g^2(0)}{E\left[n_R^2(t)\right]} = \frac{1}{\frac{N_0}{2}\int_{-\infty}^{\infty}\frac{\left|G_N(f)\right|S_{n0}(f)}{\left|C(f)\right|^2}df}$$

Nyquist only: ZF equalization...

- Problems of the ZF Equalizer:
  - noise enhancement for low channel gain (small values of $C(f)$)
  - $C(f) = 0$: ZF equalizer impossible to implement
- In general, employing exact Nyquist criterion in practical equalizers is problematic
- Develop other design criteria!

Summary

Today we discussed:

Optimal linear equalizers for linear channels 1

I. Generalized matched filter

II. GMF with Nyquist criterion

III. Nyquist only: Zero-forcing equalization

Next week: Optimal linear equalizers for linear channels II

- MMSE equalization
- Discrete-time finite-length FIR Equalizers