Statistical model describing connectivity in ad hoc networks

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Abstract

This study views the connectivity problem in ad hoc networks from a new perspective, by defining the critical transmission range for k-connectivity as a random variable and finding statistical models for this random variable when $k = 1, 2, 3$.

Introduction

The widely studied problem of connectivity in ad hoc networks has been tackled by choosing appropriate models to describe the network and deriving results analytically from these models. Typically, the transmission range has been assumed to be constant, and the most popular model for the spatial distribution of the nodes is the two-dimensional homogenous Poisson point process.

This model is partly motivated by the percolation theory. It has been shown that when disks with equal radii are generated on an infinite plane according to a Poisson process, then beyond a finite critical intensity of the process there exists almost surely a unique unbounded cluster of disks.

In one dimension, the unbounded cluster is equivalent to a connected network. However, in a recent study Dousse et al. [2] showed that the appearance of an unbounded cluster requires the domain to be infinite in two dimensions. Then again in two dimensions, an infinite cluster forming is a weaker condition than connectivity. Indeed, Philips et al. showed in [4] that with the network model of a two-dimensional Poisson point process with node density λ and transmission range r, the expected degree of a node, $\lambda\pi r^2$, must grow logarithmically with the network area in order to ensure connectivity. This renders the connectivity of random networks in the infinite plane with limited transmission range impossible.

In [1], Bettstetter and Zangl derived from properties of the Poisson process a fairly complicated analytical approximation for the probability that none of n nodes on a circular disk is isolated, by applying special geometrical analysis to zones close to the boundary where nodes are likely to have less connections than in the middle. Based on a theorem regarding the connectivity of geometric random graphs, proven by Penrose in [3], this was further approximated to give the probability that the network is connected. Even though this approximation was in a rather good agreement with simulation results, it has its shortcomings. In particular, Penrose's theorem applies when $n \to \infty$: in [1], agreement with simulation results was not demonstrated for $n < 100$.

Contributions of this study

In this study, the problem of connectivity is approached from the following probabilistic angle. Each realization of n randomly placed nodes has its threshold transmission range which is required for connectivity. This threshold range, or the *critical transmission range* $R_{\mathrm{crit}}(n)$, is therefore a random variable with a certain distribution for each n, which depends on the spatial distribution of the nodes. The definition of this random variable allows the precise quantitative study of connectivity; in several earlier studies, the empirical cumulative distribution of the threshold range has been found by pointwise simulation.

The study is extended to the stronger requirements of biconnectivity and triconnectivity. The degree of connectivity is an important property in terms of network reliability and load balancing. As in the case of connectivity, the threshold transmission range for bi- and triconnectivity can be similarly defined.

The development of algorithms for finding the threshold ranges is an integral part of this study. It turns out that in the case of simple connectivity, the threshold range is equal to the longest link in the minimum spanning tree of the nodes, as proposed by Sánchez et al. [5]. In the cases of bi- and triconnectivity, the range is found incrementally, by first finding a lower bound for it. At this point, the theorem by Penrose is an important motivation. The threshold ranges for bi- and triconnectivity are found by eliminating the articulation points and pairs, respectively, found with the lower-bound range estimate. The articulation points are easily found with the aid of a graph traversal algorithm known as the depth-first-search. However, finding the articulation pairs is notably more laborious. An algorithm based on extensions to that used for finding the articulation points is developed for the purposes of this study.

Motivated by the apparent difficulty of the analytical approach to the connectivity problem, the statistical behavior of the critical transmission range is studied through extensive simulations by utilizing the developed algorithms. This is done using the typical modeling

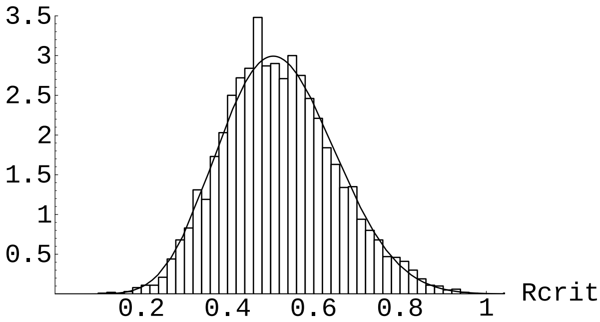

Figure 1: Histogram of R_{crit} samples and fitted Beta distribution PDF for $n = 5$ nodes in a unit square.

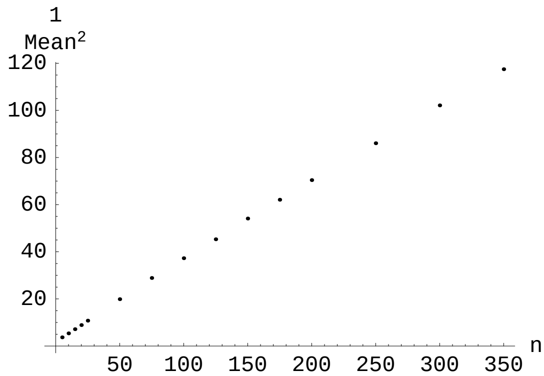

Figure 2: Squared inverses of the means of R_{crit} samples

assumptions: a chosen number of nodes uniformly distributed in a square region.

In the case of few nodes only, in the order of ten or less, the distribution is in agreement with a Beta distribution with fitted expectation and variance, scaled to the interval between zero and the length of the diagonal of the square region. The case $n = 5$ is illustrated in Figure 1.

Figure 2 shows that the squared inverse of the mean of the critical transmission range seems to grow linearly with n. Consequently, the expectation of the critical range can be described with a model of the form

$$\mathrm{E}[R_{\mathrm{crit}}(n)] = \sqrt{\frac{A}{an+b}}, \qquad (1)$$

where A is the area of the square region and a and b are the parameters of the model. What is more important, the empirical quantiles of the critical range exhibit the same behavior. The applicability of both the Beta distribution for small n and the models mentioned above extend to the cases of bi- and triconnectivity; only the parameters change.

To verify the model obtained for the threshold range for simple connectivity, its predictions are compared to results of independent simulations involving over ten times as many nodes as in this study. It turns out that this model is able to predict the results with surprising accuracy.

The asymptotic behavior of the model of the form (1) can be verified in light of the results shown in [4] by writing it in the form

$$r = 1/\sqrt{a\frac{n}{A} + \frac{b}{A}} = 1/\sqrt{a\lambda + \frac{b}{A}}, \qquad (2)$$

where r refers to the range that provides connectivity with some desired probability and λ denotes again the node density. While it can be seen from (2) that with fixed node density the required range increases with the area, the model has a finite limit as the network area tends to infinity, which is inconsistent with [4].

Conclusions

Even though finding the correct form for the statistical model for connectivity remains for future work, the presented model performs well in a wide range of settings.

The observed generalization of the statistical behavior to bi- and triconnectivity implies that k-connectivity with higher k could be predicted in a similar way, by fitting linear models to threshold range data calculated using brute-force methods but from a sufficient amount of realizations containing only small numbers of nodes.

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