

# Analytical Loss Models for MAC Protocols in Optical Ring Network Operating under a Static Traffic Load

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## Abstract

In this paper we analyse previously proposed MAC protocols for optical ring networks under static traffic conditions. The analytical models are developed for random order and round-robin transmission policies in both slotted and unslotted cases. The models predict the receiver efficiency, i.e. the fraction of time each receiver is active. The models are also verified by numerical simulations and some remarks are made for improving the performance of the protocols.

## INTRODUCTION

Optical ring network is a viable solution for metropolitan area networks (MAN). In such a network optical bursts can be used to transfer the data [1, 2]. Optical burst switching (OBS) has been proposed both for regular (e.g. ring) networks as well as general mesh networks. An optical burst consists of several concatenated packets and can be seen as an intermediate step from the wavelength routed networks (i.e. circuit switching) towards the optical packet switching. Generally in OBS the source node first sends a control packet or frame to inform the receiver (and possibly intermediate nodes) about the upcoming burst. The burst is then sent after a certain offset time without waiting for any acknowledgment from the receiver (or intermediate nodes). In this paper we will consider OBS in a ring topology.

In [3, 4] Xu et al. have studied a cost effective single fibre unidirectional OBS ring network, where each node has a dedicated fixed *home wavelength channel* for transmitting its bursts. In addition to data channels a shared control channel is used to inform the other nodes about the arriving bursts. Thus, the number of wavelength channels  $W$  is equal to  $N + 1$ , where  $N$  is the number of nodes. Furthermore each node has only one adjustable receiver. Consequently, no transmissions collide in the fibre, but as each node can listen to at most one channel at a time burst losses may occur at the receiver.

In [3, 4] several MAC protocols have been evaluated by numerical simulations. In all cases a round-robin transmission policy is used, i.e. each node maintains one transmission queue for each destination node and those transmission queues which have enough packets to form at least a mini-

mum size burst are served in round-robin fashion.

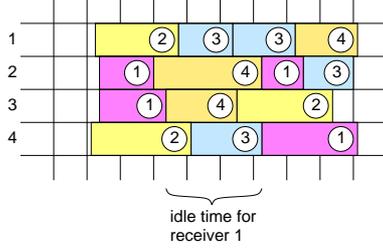
In this paper we study similar MAC protocols as proposed in [3, 4] under static traffic conditions. In particular, we consider both random order and round-robin transmission policies. When a node operates in random order policy the transmission queue to be served next is chosen randomly among the non-empty queues. In the analysis we consider an arbitrary receiver and derive formulae for the burst blocking probability and the so-called (receiver) efficiency  $\gamma$ , i.e. the proportion of the time the receiver is active. Note that in the ideal case the blocking probability is zero and the efficiency  $\gamma$  is equal to the offered load  $\rho$

A special attention is given to the performance under an extremely heavy load where each source has always bursts to be sent to all  $N - 1$  other destinations, i.e. the offered load is  $\rho = 1$ . Although this symmetric heavy traffic scenario does not hopefully exist, it gives us a lower bound on blocking probability for each MAC protocol and allows us to compare their worst case performance. Note that in an ideal case the blocking probability is zero and each receiver is busy all the time, leading to an average pairwise throughput of  $1/(N - 1)$  times the capacity of one channel. However, it will be shown that without any coordination between the nodes the actual throughput will be considerably less, i.e. about a half of that.

The rest of the paper is organized as follows. In the next two sections analytical models are developed for a unslotted protocol operating in both random order and round-robin order transmission policies and for a slotted protocol with random order transmission policy. Then we present some numerical examples, which verify the derived analytical results. In the fifth section some improvements to the protocols are proposed, and the last section contains the conclusions.

## UNSLOTTED PROTOCOL VERSION

First we consider an idealized protocol model, where transmissions of each node are independent of others. In particular, the nodes can transmit bursts at any point of time, i.e. the system is unslotted [5]. Without a loss of generality, we can consider the receiver at node 0. We assume a (quasi) static scenario where, during a some reasonably long time interval, there are  $K$  nodes sending bursts to node 0. Furthermore, each node  $j$  sends bursts to  $a_j - 1$  other nodes, i.e.  $a_j$  denotes the number of destinations channel  $j$  source has. During this



**Figure 1.** Unslotted protocol version illustrated.

*global busy period* each node sends bursts continuously to all  $a_j$  destinations in **random order** (RO) and consequently there are no idle periods in the respective channels (see Fig. 1). Note that one or more destination can be virtual, i.e. such bursts should be interpreted as real idle periods in the respective channel. Furthermore, the burst sizes are assumed to be independent and exponentially distributed,  $S \sim \text{Exp}(\mu)$ .

An idle period at node 0 receiver begins when the current burst ends and the next burst from the same source has another destination. On the other hand, the idle period ends when a new burst passing node 0 has node 0 as the destination. When the receiver is in the idle state new bursts arrive according to a Poisson process with intensities  $\lambda_j = \rho_j \cdot \mu$ , where  $\rho_j = 1/a_j$ . Thus, the total arrival rate is

$$\lambda = \sum_{j=1}^K \lambda_j = \mu \sum_{j=1}^K \rho_j = \mu\rho, \quad \text{where } \rho = \sum_{j=1}^K \rho_j,$$

and consequently,

$$E[\text{idle}] = \frac{1}{\lambda} = \frac{1}{\mu\rho}.$$

The busy period consists of one or more consecutive bursts originating from the same node and having the node 0 as the destination. Next we will deduce the average length of the busy period of node 0 receiver. The mean length of channel  $j$  busy period is  $1/(\mu \cdot (1 - \rho_j))$ . Thus,

$$E[\text{busy}] = \sum_j \frac{\rho_j}{\rho} \cdot \frac{1}{\mu \cdot (1 - \rho_j)} = \frac{1}{\mu\rho} \cdot \sum_j \frac{\rho_j}{1 - \rho_j},$$

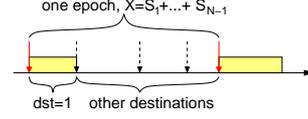
which gives us,

$$E[\text{busy}] = \beta \cdot E[\text{idle}], \quad \text{where } \beta = \sum_j \frac{\rho_j}{1 - \rho_j}. \quad (1)$$

Let  $\gamma_{\text{ns}}$  denote efficiency of the unslotted protocol, i.e. the fraction of time a receiver is active. Eq. (1) yields

$$\gamma_{\text{ns}} = \frac{E[\text{busy}]}{E[\text{busy}] + E[\text{idle}]} = \frac{\beta}{1 + \beta} = \frac{1}{1 + \beta^{-1}}. \quad (2)$$

If all the offered traffic is successfully received, the fraction of time the receiver is active is  $\gamma_{\text{max}} = \rho$ . Thus the fraction of



**Figure 2.** One epoch on each channel consists of  $N-1$  consecutive bursts.

the traffic transmitted successfully is  $1 - B = \frac{\gamma}{\gamma_{\text{max}}}$ , and consequently, the blocking probability  $B$  is

$$B = 1 - \frac{1}{\rho} \cdot \frac{\beta}{1 + \beta}, \quad \text{where } \beta = \sum_j \frac{\rho_j}{1 - \rho_j}. \quad (3)$$

## Symmetric Cases

Consider first a case where each  $a_j$  is some constant, i.e.  $\rho_j = \rho/K$ . It is easy to see that the system asymptotically converges to an  $M/M/1/1$ -system,  $\lim_{K \rightarrow \infty} B = \rho/(1 + \rho)$ .

Consider next a case where each node constantly sends bursts to all other nodes, i.e.  $K = N - 1$ . In this case the offered load  $\rho = 1$  and the relationship between  $\gamma$  and  $B$  is simply  $B = 1 - \gamma$ . Thus  $\beta = (N - 1)/(N - 2)$  which yields,

$$\gamma_{\text{ns}}(N) = \frac{N - 1}{2N - 3}, \quad \text{or, } B = \frac{N - 2}{2N - 3}.$$

Thus, blocking probability  $B$  increases from 0 to 0.5 as the number of nodes  $N$  goes from 2 to infinity.

## Round-Robin Transmissions

Assume that each node maintains one queue for each destination. The previous model assumed that the destination of each burst is randomly picked independently of others, i.e. each time a queue to be served next is picked randomly. Another alternative is to serve queues in round-robin fashion (RR). For simplicity let us consider the symmetric case with  $K = a_j = N - 1$ . One full period on each channel starts with a transmission to node 0 and ends when the next burst to node 0 is about to be sent (see. Fig. 2). As before, we assume that burst lengths are independent and exponentially distributed with parameter  $\mu$ . Hence, the busy period is clearly exponentially distributed with parameter  $\mu$ ,

$$E[\text{busy}] = E[S] = 1/\mu.$$

The idle time is harder to deduce. The system state consists of the states of  $N - 1$  independent channels defined by the destination of the bursts in progress. At the end of a busy period a transmission has just ended on one channel, while the other  $N - 2$  channels can be in any phase. We make an assumption that the state of the other channels at the end of a busy period obeys the equilibrium distribution. This is not exactly true,<sup>1</sup> but serves as a good approximation.

<sup>1</sup>The fact that an active burst ends at a certain time gives some information about the phases of the other channels. Thus, the end point of a burst does not represent a random point of time.

Generally the time to the next arrival involves the determination of the residual times of  $N - 2$  random variables corresponding to the times until the next burst to node 0 begins on the other channels. To be exact, one needs to find the minimum of those  $N - 2$  residual times and one full period, corresponding to the channel in which the burst just ended. Let  $X$  denote the length of one period,  $X = S_1 + \dots + S_{N-1}$ , and  $G(t) = P\{X > t\}$ . Then,

$$G(t) = \int_t^\infty \frac{\mu^{N-1}}{(N-2)!} \cdot x^{N-2} \cdot e^{-\mu x} dx = e^{-\mu t} \cdot \sum_{i=0}^{N-2} \frac{(\mu t)^i}{i!}.$$

Generally the tail distribution of the residual times is

$$G_{\text{res}}(t) = \frac{1}{E[X]} \cdot \int_t^\infty G(x) dx.$$

The tail probability function of the time to the next arrival, i.e. the probability that the idle time is greater than  $t$ , is,

$$\begin{aligned} P\{\text{idle} > t\} &= G(t) \cdot (G_{\text{res}}(t))^{N-2} \\ &= \frac{1}{E[X]^{N-2}} \cdot G(t) \cdot \left( \int_t^\infty G(x) dx \right)^{N-2}, \end{aligned}$$

which yields,

$$\begin{aligned} E[\text{idle}] &= \int_0^\infty P\{\text{idle} > t\} dt \\ &= \frac{1}{E[X]^{N-2}} \int_0^\infty G(t) \cdot \left( \int_t^\infty G(x) dx \right)^{N-2} dt. \quad (4) \end{aligned}$$

For efficiency  $\gamma_{\text{ns,rr}}$  one obtains

$$\gamma_{\text{ns,rr}} \approx \frac{E[\text{busy}]}{E[\text{busy}] + E[\text{idle}]} \quad (5)$$

### Minimum of Gamma Distributions

In the case of gamma distribution one can avoid the evaluation of the integrals in Eq. (4) by using the memoryless property of exponential distribution [6, 7]. Thus, we need to determine the mean value of the minimum of  $N$  random variables  $G_j \sim \text{Gamma}(a_j, \mu)$ , where  $j = 1, \dots, N$ , and  $a_j$  are some integer constants. In other words, determine  $E[\min_j G_j]$ . Next we re-formulate the problem as a Markov process and then describe a recursive solution for it. Consider a system where  $N$  customers are served concurrently and the  $i$ th customer's service time consists of  $a_i$  exponentially distributed phases. The quantity we are interested in is the average time until the first customer leaves the system. As long as no customer has left the system the instants of transitions from one phase to another constitute a Poisson process with a constant intensity of  $N \cdot \mu$ . Thus, it is sufficient to determine the mean number of transitions to the first departure. To this end we

consider the respective embedded Markov chain and aggregate the state space as follows. Let  $x_j$ ,  $j = 1, \dots$ , denote the number of customers having  $j$  phases left to the end of service and  $x_0$  the number of customers which already have left the system. Thus, in each transition one customer moves from some  $x_j$  to  $x_{j-1}$  and initially we have,

$$x_j = \sum_{i=1}^N 1_{a_i=j} \quad \text{and} \quad x_0 = 0.$$

Let  $s(\mathbf{x}^*)$  denote the mean number of steps to the first departure from state  $\mathbf{x}^* = (x_0, x_1, \dots)$ . Due to the memoryless property it holds that

$$s(\mathbf{x}^*) = \begin{cases} 0, & \text{if } x_0 > 0, \\ 1 + \sum_j \frac{x_j}{N} \cdot s(\mathbf{x}^* - \mathbf{e}_{j+1} + \mathbf{e}_j), & \text{if } x_0 = 0, \end{cases} \quad (6)$$

where  $\mathbf{e}_j$  is a vector with  $j$ th component equal to 1 while the rest are zeroes. For example, for  $x_1 = 3$  and  $x_2 = 1$ ,

$$\begin{aligned} s(\{0, 3, 1\}) &= 1 + \frac{3}{4} \cdot s(\{1, 2, 1\}) + \frac{1}{4} \cdot s(\{0, 4, 0\}) \\ &= 1 + 0 + \frac{1}{4} \left( 1 + \frac{4}{4} s(\{1, 3, 0\}) \right) = \frac{5}{4}. \end{aligned}$$

Finally, the mean time to the first departure is simply,

$$m(\mathbf{x}) = \frac{1}{\mu N} \cdot s(\{0 \mathbf{x}\}), \quad \text{where } \mathbf{x} = (x_1, x_2, \dots). \quad (7)$$

A similar derivation could be applied to the discrete time case where the  $j$ th customer's service time consists of  $a_j$  geometrically distributed time intervals. However, the recursion becomes more complex as more than one service period may end at the same time.

### Recursive Algorithm for Idle Time

Next we return to the original problem of determining the mean idle time of receiver in case of round-robin transmission policy. First, enumerate the states of each channel with numbers  $1, \dots, N - 1$ , so that state  $j$  means that the  $j$ th burst in the channel will have node 0 as the destination. For instance, in state  $N - 1$  the channel is currently transmitting a burst to node 0 and it takes  $N - 1$  epochs until the next burst to node 0 starts. At the start of the idle period the channel the transmission of which just ended is in state  $N - 2$  and the other  $N - 2$  channels were assumed to obey the nodeary distribution, i.e. each of them is equally likely to be in any state  $1, \dots, N - 1$ .

Similarly as in the previous section we use the aggregate state space. Let  $x_j$  denote the number of channels being in state  $j$ , so that the vector  $\mathbf{x} = (x_1, x_2, \dots, x_{N-1})$  describes the essential information about the state of the system. Consider next the other  $N - 2$  channels. The stationary

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**Algorithm 1** Unslotted protocol with round-robin policy
 

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Let  $X$  denote the state space of  $N - 2$  channels,

$$X = \{(x_1, \dots, x_{N-1}) : x_j \geq 0 \text{ and } \sum_j x_j = N - 2\}.$$

Let  $p(\mathbf{x})$  denote the probability of the state  $\mathbf{x} \in X$ ,

$$p(\mathbf{x}) = \frac{1}{(N-1)^{N-2}} \prod_i \binom{N - \sum_{j=1}^{i-1} x_j}{x_i}.$$

Calculate the mean idle period using Eq. (7),

$$E[\text{idle}] = \sum_{\mathbf{x} \in X} p(\mathbf{x}) \cdot m(\mathbf{x} + \mathbf{e}_{N-2}).$$

Calculate the estimate for the efficiency using Eq. (5).

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distribution of  $N - 2$  channels in the aggregated state space, denoted by  $X$ , is straightforward to obtain. As each channel was equally likely to be in any state  $1, \dots, N - 1$ , the situation is equivalent to placing  $N - 2$  balls (channels) to  $N - 1$  urns (states). Thus, this is a classical combinatorial setup and the total number of states in  $X$  is simply  $\|X\| = \binom{N-4}{N-2}$ .

From this point onwards the solution is straightforward and is presented in Algorithm 1. Note that adding  $\mathbf{e}_{N-2}$  in step 3 takes into account the channel whose reception just ended. The algorithm can be implemented efficiently using recursive functions, where both the states and the respective probabilities are obtained at the same time.

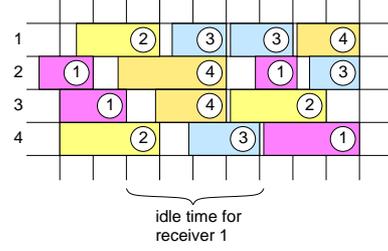
### SLOTTED PROTOCOL VERSION

The previous model neglected the fact that in the OBS ring network, in addition to data bursts, also a certain number of control frames circulate (see, e.g. [3, 4]). The control frames are separated by a fixed time intervals  $\Delta T$ ,

$$\Delta T = \frac{D + N \cdot T_\mu}{H},$$

where  $D$  is the total propagation delay around the ring,  $N$  the number of nodes,  $T_\mu$  the control frame processing time and  $H$  the total number of control frames. The receiving node is acknowledged of arriving burst in the previous control frame so that bursts always arrive exactly  $T_\mu + T_s$  time units after the corresponding control frame, where  $T_s$  is the so-called switching delay reserved for the destination node to setup the reception. For our analysis it is sufficient to consider the control frames, which arrive, as described above, at constant intervals resulting a slotted model as illustrated in Fig. 3.

Similarly as before, assume **random order** transmission policy and that during some considerably long time interval node  $j$ ,  $j = 1, \dots, K$ , has  $a_j$  destinations of which one is node 0 we are interested in. In particular, we assume that the burst



**Figure 3.** Slotted version of MAC protocol illustrated.

length distribution is

$$S = (X - U) \cdot \Delta T,$$

where  $X$  is a geometrically distributed random variable with parameter  $q$  corresponding to the number of slots burst reserves, and  $U$  is uniformly distributed random variable in the interval  $[0, 1]$  and corresponds to the residual time to the start of the next slot. Thus, an active burst ends during the next time slot with probability  $q$  and the mean number of slots in a burst is

$$E[X] = E[\text{burst length in slots}] = 1/q,$$

where  $q = 2/(2 \cdot E[S] + 1)$ . Once a burst ends at receiver the next possible burst can start in the next slot. Let  $Y$  be the number of idle slots in receive,  $Y = 0, 1, 2, \dots$ . Assume that a reception of a burst on channel  $j$  ends. Then the probability that no new burst starts during the following slot is

$$\left(1 - \frac{1}{a_j}\right) \cdot \prod_{i \neq j} \left(1 - \frac{q}{a_i}\right).$$

Generally, letting  $Y$  denote the idle time, it holds that

$$\begin{aligned} P\{Y > 0 | \text{ch. } j \text{ was the last}\} &= \left(1 - \frac{1}{a_j}\right) \cdot \prod_{i \neq j} \left(1 - \frac{q}{a_i}\right), \\ P\{Y \geq k + 1 | Y \geq k\} &= \prod_i \left(1 - \frac{q}{a_i}\right). \end{aligned}$$

Hence,

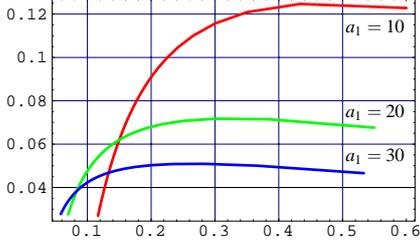
$$\begin{aligned} P\{Y = k | \text{channel } j \text{ was the last}\} &= \begin{cases} \beta_0^{(j)}, & \text{when } k = 0, \\ \left(1 - \beta_0^{(j)}\right) \cdot \beta^{k-1} \cdot (1 - \beta), & \text{when } k > 0, \end{cases} \end{aligned}$$

where

$$\beta = \prod_i \left(1 - \frac{q}{a_i}\right) \quad \text{and} \quad \beta_0^{(j)} = 1 - \frac{a_j - 1}{a_j - q} \cdot \beta.$$

Thus, the mean idle period  $E[Y]$  is

$$E[Y] = \frac{1}{1 - \beta} \cdot \sum_{j=1}^K \left(1 - \beta_0^{(j)}\right) \cdot P\{\text{channel } j \text{ was the last}\}.$$



**Figure 4.** Throughput of two active channels as a function of offered total load  $\rho = 1/a_1 + 1/a_2$  for  $a_1 = 10, 20, 30$ .

In particular,  $1 - \beta_0^{(j)} \approx \beta$  when  $a_j \gg 1$  and we get

$$E[Y] \approx \frac{\beta}{1 - \beta}. \quad (8)$$

Similarly as in the unslotted case we get for the efficiency that

$$\gamma_s = \frac{E[S]}{E[X] + E[Y]} = \frac{1/q - 1/2}{1/q + E[Y]}. \quad (9)$$

Note that when determining the blocking probability we should consider only the slots and neglect the residual time  $U$ . Thus, *slot efficiency* is

$$\gamma_s^* = \frac{E[X]}{E[X] + E[Y]} = \frac{1/q}{1/q + E[Y]}.$$

### Symmetric Cases

For a large number of active sources  $K$  with  $a_j \approx \alpha \cdot K$ ,  $\forall j$ , the offered load is  $\rho = 1/\alpha$  and  $\beta \approx e^{-\rho q}$ . Thus the mean idle period, given by Eq. (8), becomes

$$E[Y] \approx \frac{1}{e^{\rho q} - 1},$$

and the blocking probability  $B$  is consequently,

$$B \approx \frac{\rho - \gamma_s^*}{\rho} = 1 - \frac{1}{\rho} \cdot \frac{1/q}{1/q + 1/(e^{\rho q} - 1)},$$

which can be rewritten as (cf. Eq. (3)),

$$B \approx 1 - \frac{1}{\rho} \cdot \frac{z}{1+z}, \quad \text{where } z = \frac{e^{\rho q} - 1}{q}. \quad (10)$$

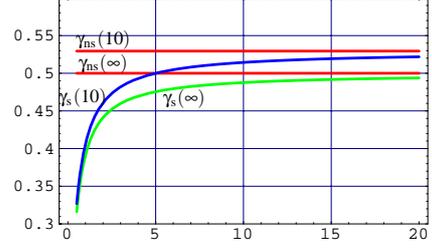
Note also that  $z \approx \rho$  when  $\rho q \ll 1$ , i.e.

$$B \approx \frac{\rho}{1+\rho}, \quad \text{when } \rho q \ll 1. \quad (11)$$

Hence, Eqs. (3) and (10) are asymptotically equivalent.

For the symmetric heavy traffic load case each node sends continuously bursts to all other nodes, i.e.  $K = a_j = N - 1$  and

$$\beta = \left(1 - \frac{q}{N-1}\right)^{N-1}$$



**Figure 5.** Efficiency of the unslotted (ns) and slotted (s) protocols for  $N = 10$  and  $N = \infty$  as a function of  $E[S]$ .

Especially for  $N \gg 1$  we have  $a_j = K \gg 1$ , and hence

$$E[Y] \approx \frac{1}{e^q - 1},$$

and consequently the efficiency is

$$\gamma_s(\infty) = \frac{E[S]}{E[X] + E[Y]} = \frac{1/q - 1/2}{1/q + 1/(e^q - 1)}. \quad (12)$$

From Eq. (12) it is easy to see (the obvious fact) that  $\gamma_s(\infty) \leq \frac{1}{2}$ , and hence the slotted version is never better than the unslotted one. Furthermore, for large values of  $1/q$ , corresponding to very long burst sizes, we have  $\gamma_s(\infty) \rightarrow 1/2$ , i.e. at the limit  $E[S] \rightarrow \infty$  the efficiencies converge to the same point,  $\gamma_s(\infty) = \gamma_{ns}(\infty) = 1/2$ .

## NUMERICAL EXAMPLES

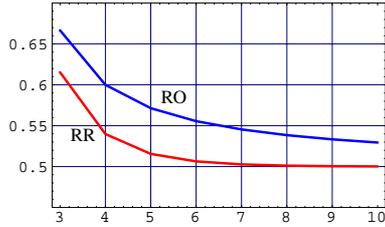
### Asymmetric Traffic

For the numerical examples we consider first an asymmetric traffic with two active channels. In Fig. 4 the blocking probability  $B$  given by (3) is plotted for three different sets. On the  $x$ -axis is the offered load  $\rho = \frac{1}{a_1} + \frac{1}{a_2}$  and on the  $y$ -axis is the blocking probability. Each curve represents a situation  $\mathbf{a} = \{a_1, a_2\}$ , where parameter  $a_2 = x$  is varied,  $x = 2, 3, \dots$ , while the parameter  $a_1$  is kept constant,  $a_1 = 10, 20, 30$  (from top to bottom). In each case the curve is convex and has a unique maximum point. Furthermore, the bigger the difference between the channel loads is, the lower the overall blocking probability is. Hence, it is advantageous to have inhomogenous traffic load.

### Symmetric Heavy Traffic

Next we consider the symmetric cases where each node sends constantly bursts to all other nodes, i.e.  $\rho = 1$ . In Fig. 5 the efficiency given by Eqs. (2) and (9) is depicted as a function of mean burst size for  $N = 10$  and  $N = \infty$ . The constant lines represent the unslotted protocol version ( $\gamma_{ns}$ ) and the lower increasing curves the slotted version ( $\gamma_s$ ). In both cases the upper curve corresponds to the case  $N = 10$ .

From Fig. 5 it can be seen, as expected, that the slotted version is generally less efficient than the unslotted one. When



**Figure 6.** Efficiency of unslotted MAC protocol with random order (upper curve) and round-robin (lower curve) transmission policies as function of the number of nodes  $N$ .

the mean burst size is roughly 5 times the control frame interarrival time  $\Delta T$  or more, the performance of the slotted version comes quite close to that of the unslotted one. By increasing the mean burst size one can increase the efficiency arbitrarily close to that of the unslotted version. However, this also leads to an increase in burstification delay and thus cannot be made arbitrarily high.

In Fig. 6 the efficiency of the unslotted version is depicted for random order (upper curve) and round-robin (lower curve) transmissions as a function the number of nodes  $N$ . The random order policy is clearly better when the number of nodes is moderate. In the limit  $N \rightarrow \infty$  both versions will converge to the same point,  $\gamma = 0.5$ .

## Simulated Results

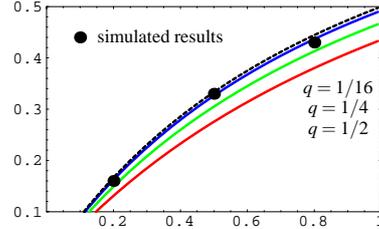
The symmetric heavy traffic scenario was also analyzed by numerical simulations. For both the slotted and unslotted version we considered the two transmission policies, i.e. random order and round-robin. The ring consisted of  $N = 10$  nodes and the mean burst length  $E[S]$  was chosen to be 4 time slots. Thus, in the unslotted version  $\mu = 1/4$ , while in the slotted version  $q = 2/9$ . Consequently, the mean number of slots reserved by a single burst was 4.5.

The simulation results together with the respective analytical results are presented in Table 1. It can be noted that the analytical formulae are accurate. Furthermore, the introduction of round-robin transmission policy does not seem to degrade the performance much in this case.

We also simulated another variant of the slotted protocol version, where the receiver chooses the longest arriving burst in case of concurrent arrivals. With this modification the efficiency of the random order protocol improved from 0.490 to 0.504, and for the round-robin version from 0.478 to 0.492.

**Table 1.** Simulation results with  $N = 10$  and  $E[S] = 4$ .

scenario		unslotted	slotted
RO	analytical	0.529	0.488
RO	simulated	$0.531 \pm 0.002$	$0.490 \pm 0.003$
RR	analytical	0.500	—
RR	simulated	$0.514 \pm 0.002$	$0.478 \pm 0.001$



**Figure 7.** Blocking probability given by Eq. (10) for  $1/q = 2, 4, 16$  (solid curves) and an upper bound (dashed curve) given by Eq. (11) as a function of total offered load  $\rho$ .

## Realistic Scenario

The final example is a realistic scenario where  $\rho < 1$ . The simulation parameters were as follows:

- $N = 10$  nodes and  $H = 30$  control frames
- data channel link speed of 2.5 Gbps,
- burst sizes from 16kB to 112kB (i.e.  $S \sim \text{Exp}(\mu)$ ),
- $T_\mu = 13 \mu\text{s}$  and  $T_s = 1 \mu\text{s}$  (processing/switching time)

Consequently,  $\Delta T = T_\mu \approx 13 \mu\text{s}$ , and the mean burst size was about  $16 \cdot \Delta T$ , i.e. 16 time slots. In Fig. 7 the blocking probability  $B$  given by Eq. (10) is plotted for three different mean burst sizes. The mean burst size does not seem to affect much on the blocking probability. The three dots in the figure at  $\rho = 0.2, 0.5$  and  $0.8$  correspond to actual simulated results. From Fig. 7 it can be seen that the respective curve matches well with the simulated results.

## SYSTEM PARAMETERS

Feasible control interarrival times,  $\Delta T$ , are bounded by the processing time  $T_\mu$ , i.e.  $\Delta T > T_\mu$ . Thus, the shorter the processing time is, more control frames can be used and the better performance we can expect.

Assuming that the mean burst size is moderate the slotted MAC protocol model describes the system appropriately. Note that in the limit  $\Delta T/E[S] \rightarrow 0$  one gets the unslotted protocol version. Based on the previous observations one can improve the standard MAC protocol without introducing any new complex mechanisms for coordination of the transmissions between different nodes. The following modifications will not bring any dramatic improvement in the performance, but nonetheless, they are inexpensive to implement.

## Burstification

A typical burstification process sends a burst after a time  $T_b$  from the arrival of the first packet, or when the queue length grows above a certain limit  $S_{\text{thres}}$ , whichever occurs first.

It is clearly advantageous to have a long mean burst size, which minimizes the effect from the burst lengths not being integer multiples of the control frame interarrival times. However, long bursts lead to a longer burstification delay, which

means that packets also experience a longer transmission delay in the network. Thus, deciding on the burst length is a compromise between the throughput and the mean transmission delay and one should choose the burstification process so that the mean burst length is the maximum for which the mean delay still stays under a certain limit given by the design criteria. From Fig. 5 one can see that a mean burst length of  $5 \cdot \Delta T$  or more gives a reasonable throughput.

Furthermore, it is beneficial to send bursts the lengths of which are integer multiples of the control frame interarrival times  $\Delta T$ , i.e.  $S = X \cdot \Delta T$  where  $X = 1, 2, \dots$ . However, in practice this is not possible unless the arriving packets are split into several bursts, which would require complex storing and packet reassembly capabilities in the receiver.

We propose additional rules for burstification process so that the burst lengths become close to integer multiples of  $\Delta T$  while not exceeding it. In particular, upon sending a burst node adds as many packets to burst as possible without exceeding the next multiple of  $\Delta T$ , or alternatively leaves some packets out. With a uniform burst length distribution half of the last slot capacity is wasted on average. Thus, the possible improvement in efficiency  $\gamma_s$  cannot be more than *half a slot time*. In the previous example case ( $N = 10$  and  $E[S] = 4$ ) this corresponds to improvement from 0.49 to 0.53 in efficiency, i.e. about 8% decrease in the blocking probability.

### Transmission Policy

Under a heavy load the round-robin policy decreases the system performance. However, under a normal traffic load it guarantees a close to constant transmission delay, which is generally beneficial for example to TCP flows. Thus, an ideal solution would dynamically switch from round-robin policy to random policy when the system load increases. The local queue lengths or burst blocking probability could be used to decide on the current traffic conditions.

### Contention at Receiver

When contention occurs at the receiver the proposed protocols choose randomly one of the bursts. We propose that the receiver, instead of choosing a random burst, chooses the longest one. The benefit from this is obvious. In the example case ( $N = 10$  and  $E[S] = 4$ ) this modification improves the efficiency from 0.49 to 0.50.

### CONCLUSIONS

In this paper we have derived analytical formulae for blocking probability and so-called efficiency in an optical burst switching ring network operating under two different MAC protocols. By efficiency we mean the proportion of time a given receiver is active, which is clearly related to the throughput from the receiver point of view. The analytical

models were derived for static traffic load conditions, where a certain number of nodes are continuously sending bursts to a given node as well as to some other destinations. During this *global busy period* it was assumed that each node has constantly bursts in queues ready to be sent. As a special case a heavy traffic scenario was considered, where each node sends bursts to all other nodes constantly. This heavy load scenario serves as a worst case performance scenario for comparing different MAC protocols and also gives an upper bound for efficiency in case of uniform traffic load.

The analytical models and simulation results all showed similar performance figures. With a low and moderate traffic loads the performance figures predicted by the analytical models matched well with the simulated results. Furthermore, in an extremely heavy load scenario the overall efficiency was of the order of 0.48 – 0.53, corresponding to a blocking probability of 47 – 52%. In order to reach a higher efficiency (i.e. a lower blocking probability) one needs to consider some kind of global coordination of transmissions, e.g. as suggested in [3, 4].

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