On Optimality of Single-Path Routes in Massively Dense Wireless Multi-Hop Networks

Esa Hyytiä
Telecommunications Research Center Vienna
Donau-City Straße 1, 1220 Vienna, Austria
esa.hyytia@ftw.at

Jorma Virtamo
Networking Laboratory
Helsinki University of Technology, Finland
jorma.virtamo@tkk.fi

ABSTRACT

We consider the load balancing problem in large wireless multi-hop networks, often referred to as massively dense wireless multi-hop networks. A network is considered to be massively dense if there are nodes practically everywhere and a typical distance between two nodes is much larger than the transmission range necessitating communication over a large number of hops. The task is to choose the routes in such a way that the maximum relayed traffic load in the network is minimized. In fixed networks the multi-path routes generally yield a lower congestion and thus allow higher throughput. In contrast, we show that in the case of massively dense wireless multi-hop networks the optimal load balancing can be achieved by single-path routing. In particular, we show how any given multi-path routing can be transformed to a single-path routing with at least the same level of performance. The concepts are illustrated by numerical examples where the network nodes are assumed to reside inside a unit disk with uniform traffic demands. The shortest path routes, corresponding to straight line segments, yield a maximum traffic load of 0.637, whereas the single-path routes obtained by numerical optimization yield 0.343, corresponding to 46% reduction in the traffic load.

Categories and Subject Descriptors

C.2.2 [**Computer-Communication Networks**]: routing protocols

General Terms

Algorithms, Performance, Theory

Keywords

wireless multi-hop network, multi-path, single-path, load balancing

1. INTRODUCTION

Load balancing problem in fixed networks is a well-known problem for which several formulations have been proposed. Typically one is asked to find such routes that minimize the maximum link load. In general, lower congestion can be obtained by dividing the traffic flows to several routes, i.e., by using multi-path routes. Typically the problem description lends itself to an efficient formulation as an LP-problem, see, e.g., [1] and Appendix. The approach is generic and has been applied to different kinds of networks ranging from packet switching to lightpath routed optical networks.

In this paper, we consider the load balancing problem in a wireless multi-hop network in the setting of a so-called massively dense multi-hop network. Massively dense multi-hop network means that 1) there are nodes practically everywhere, and 2) that a typical path between two nodes consists of a large number of hops, i.e., the transmission range is several orders of magnitude smaller than the diameter of the network. In this setting there is a strong separation between the microscopic level, corresponding to the immediate neighbourhood a given node, and the macroscopic level, corresponding to the end-to-end connections. Note that in the considered type of networks the traffic load consists almost solely of the relay traffic. At the microscopic level the nodes are simply concerned about forwarding a given packet to the direction defined by the chosen routing. In practice this means, roughly speaking, that locally a packet is forwarded to the furthest reachable node in that direction. At the macroscopic level one is concerned about the end-to-end paths and the assumption of the strong separation between the different scales justifies describing the paths as smooth continuous curves [2–8]. In the present paper we focus on studying the optimal paths at the macroscopic level.

In a similar context both Pham et al., in [9], and Ganjali et al., in [10], have considered the possible gain from using K shortest paths instead of one. By using approximative modelling techniques Pham et al. argue that the use of multi-path routing always results in improvement to throughput. However, the results by Ganjali et al., based on modelling the K shortest paths as a rectangle between the node pairs, suggest that this is not always the case, unless a huge number of multiple paths are allowed for each pair of nodes. This is due to the fact that in the limit of massively dense network any finite number of paths K tends to be (close to) a line segment between the nodes. Finally it is concluded that in order to achieve better load balancing one needs to find such routes which push the traffic away from the center of the network, i.e., by using not only the (K) shortest paths.

In [11, 12], we have considered the load balancing problem in dense wireless multi-hop networks. In contrast to [9, 10], we focused on single-path routing with curvilinear paths, and noted that several single-path routes can straightforwardly be combined to multi-path routing by randomly choosing one of the single-path routes for each packet. In the present paper, we show that with respect to load balancing problem the optimal solution can always be achieved by properly chosen single-path routing. This new result is in strike contrast to fixed networks, where restriction to single-path routes can severely limit the performance of the network. By single-path routing we obviously refer to the routes at the macroscopic level.

The rest of the paper is organized as follows. In Section 2 we introduce the notation and define the load balancing problem, and then prove that the optimal solution can be achieved by a single-path routing. In Section 3 we illustrate the framework and the new results by means of numerical examples, and Section 4 contains the conclusions.

2. LOAD BALANCING PROBLEM

To start with we first need to define the traffic that is offered to the network located in some area denoted by $\mathcal{A}$. In our setting the nodes form a continuum and thus it is convenient to define the traffic demands as densities:

Definition 1 (traffic demand density) *The rate of flow of packets from a differential area element dA about $\mathbf{x}$ to a differential area element dA about $\mathbf{r}$ is $\lambda(\mathbf{x}, \mathbf{r}) \cdot dA^2$, where $\lambda(\mathbf{x}, \mathbf{r})$ is called the traffic demand density [pkts/s/m^4].*

As part of our earlier work, in [11, 12], we have defined the load balancing problem in dense multi-hop networks as a minmax problem for the scalar packet flux. Scalar packet flux in turn is defined in terms of so-called angular packet flux[1] (see Fig. 1):

Definition 2 (angular flux) *Angular flux of packets at $\mathbf{x}$ in direction θ, denoted by $\varphi(\mathbf{x}, \theta)$, is equal to the rate [1/s/m/rad] at which packets flow in the angle interval $(\theta, \theta + d\theta)$ across a small line segment of the length ds perpendicular to direction θ at point $\mathbf{x}$ divided by $ds \cdot d\theta$ in the limit when $ds \to 0$ and $d\theta \to 0$.*

Definition 3 (scalar flux) *Scalar flux of packets [1/s/m] at $\mathbf{x}$ is given by*

$$\Phi(\mathbf{x}) = \Phi(\mathcal{P}, \mathbf{x}) = \int_0^{2\pi} \varphi(\mathcal{P}, \mathbf{x}, \theta)\, d\theta. \tag{1}$$

The load balancing problem in the context of massively dense multi-hop network is stated as follows [11, 12]:

Definition 4 (load balancing problem) *Find the set of paths $\mathcal{P}$ which minimizes the maximum scalar flux,*

$$\arg\min_{\mathcal{P}} \max_{\mathbf{x}} \Phi(\mathbf{x}). \tag{2}$$

In order to analyze the multi-path routing we consider the packet flows having some fixed destination $\mathbf{r} \in \mathcal{A}$. To this end, we need some additional definitions.

Definition 5 (angular d-flux density) *Angular d-flux density, denoted by $\varphi(\mathbf{x}, \theta; \mathbf{r})$ [1/s/m^3/rad], is equal to the angular flux $\varphi(\mathbf{x}, \theta)$ resulting from the packets having their final destination in small area dA about $\mathbf{r}$ divided by dA in the limit when $dA \to 0$.*

Thus, by definition,

$$\varphi(\mathbf{x}, \theta) = \int_{\mathcal{A}} \varphi(\mathbf{x}, \theta; \mathbf{r})\, d^2\mathbf{r}.$$

[1]Note that in [11, 12] angle θ denotes the angle from which packets arrive, whereas in the present paper it is more convenient to define θ as the direction to which packets are moving, i.e., there is a difference of π between these two conventions.

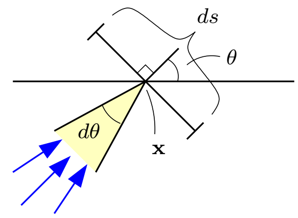

Figure 1: Angular flux $\varphi(\mathbf{x}, \theta)$ is the rate of packets crossing a small line segment ds in angle $(\phi, \phi + d\phi)$ divided by $d\phi \cdot ds$ at the limit $d\phi, ds \to 0$.

Definition 6 (d-flow intensity) *Destination flow intensity or for short d-flow intensity of packets at $\mathbf{x}$ having destination $\mathbf{r}$, denoted by $\mathbf{J}(\mathbf{x}, \mathbf{r})$ [1/s/m^3], is equal to*

$$\mathbf{J}(\mathbf{x}, \mathbf{r}) = \int_0^{2\pi} \varphi(\mathbf{x}, \theta; \mathbf{r})\, \mathbf{e}_\theta\, d\theta, \tag{3}$$

where $\mathbf{e}_\theta$ is the unit vector in direction θ.

We note that for a given destination $\mathbf{r}$ the d-flow intensity $\mathbf{J}(\mathbf{x}, \mathbf{r})$ corresponds to a vector field in $\mathbb{R}^2$. Moreover, e.g., in a mesh network with a single gateway node there is essentially only one destination in the network, and thus it is sufficient to consider d-flow intensity towards the location of the gateway. From the load balancing point of view, this scenario, however, is trivial.

2.1 Single-path routing

The most obvious choice for paths are the shortest paths, which in this context correspond to straight line segments between the source and destination. This, however, tends to concentrate too much traffic to the center of the network. Thus, in order to avoid congestion one needs to bend some routes away from the center. A convenient way to define the single-path routes is as follows:

Definition 7 (single-path routing) *With single-path routing the direction ϑ at which a given packet is forwarded at $\mathbf{x}$ is defined solely by the destination $\mathbf{r}$ of the packet, $\vartheta(\mathbf{x}, \mathbf{r}) : \mathbb{R}^2 \times \mathbb{R}^2 \to [0, 2\pi)$.*

Consequently, the end-to-end paths $\mathcal{P}$ are defined by ϑ, $\mathcal{P} = \mathcal{P}(\vartheta)$. In [11, 12] explicit expressions for calculating the scalar packet flux for arbitrary single-path routing is given. Note that ϑ-forwarding rule is actually a stronger requirement than just a single-path between any given pair of locations because from ϑ-forwarding rule it follows that all the other nodes along the same path also use the same (remaining) path for the packets going to the same destination. In fact, in [12] this distinction is made explicitly and the forwarding rule according to Def. 7 is referred to as *destination based forwarding*. In the present paper, we show that the optimal solution with regard to load balancing problem can be achieved with a destination based forwarding rule according to Def. 7, and thus we do not make further distinction between more relaxed definitions of single-path routes.

Additionally the communication between two locations may use the same path in both directions:

Definition 8 (bidirectionality) *Paths are bidirectional if $p(\mathbf{r}_2, \mathbf{r}_1)$ is $p(\mathbf{r}_1, \mathbf{r}_2)$ in reverse direction for all $\mathbf{r}_1, \mathbf{r}_2 \in \mathcal{A}$.*

Note that a flow on a given path contributes to the scalar flux at any point on the path by an amount equal to the absolute size of the

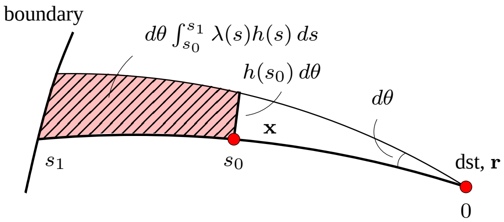

Figure 2: Expression for d-flow $\mathbf{J}(\mathbf{x})$ with arbitrary single-path routes.

flow, no matter what the direction of the flow is. Thus, allowing a different return path is, from the load balancing point of view, is in effect equivalent to allowing two paths between each pair of locations (assuming that there is traffic flowing in both directions).

It is straightforward to see that with single-path routing we have

$$\vartheta(\mathbf{x}, \mathbf{r}) = \arg \mathbf{J}(\mathbf{x}, \mathbf{r}), \tag{4}$$

and

$$\varphi(\mathbf{x}, \theta; \mathbf{r}) = \delta(\theta - \vartheta(\mathbf{x}, \mathbf{r})) \cdot |\mathbf{J}(\mathbf{x}, \mathbf{r})|. \tag{5}$$

where $\arg \mathbf{J}(\mathbf{x}, \mathbf{r})$ denotes the angle of direction of vector $\mathbf{J}(\mathbf{x}, \mathbf{r})$, and $\delta(\cdot)$ is Dirac's delta-function. Moreover, for scalar packet flux with single-path routing we have

$$\Phi(\mathbf{x}) = \int_0^{2\pi} \varphi(\mathbf{x}, \theta)\, d\theta = \int_{\mathcal{A}} |\mathbf{J}(\mathbf{x}, \mathbf{r})|\, d^2\mathbf{r}. \tag{6}$$

In general, for an arbitrary domain with single-path routes, there are paths arriving to a given destination $\mathbf{r}$ from all angles θ, $\theta \in [0, 2\pi)$. In particular, let $\mathbf{p}(s; \mathbf{r}, \theta)$ denote the path to $\mathbf{r}$ arriving at the angle θ where parameter s corresponds to the distance to $\mathbf{r}$ along the path, i.e., we assume that there is a one-to-one correspondence between paths and arriving angles θ at the destination $\mathbf{r}$. The situation is illustrated in Fig. 2. (Note that this is an additional constraint which we make here for the notational simplicity. Note also that there exist valid though somewhat pathological routes which do not satisfy this assumption, e.g., the so-called radial ring paths explained later in Section 3.) However, in this case the magnitude of d-flow at $\mathbf{x}$ to destination $\mathbf{r}$ can be obtained by evaluating a line integral,

$$|\mathbf{J}(\mathbf{x}, \mathbf{r})| = \frac{1}{h(s_0)} \int_{s_0}^{s_1} \lambda(s) h(s)\; ds, \tag{7}$$

where $\lambda(s) = h(s; \mathbf{x}, \mathbf{r})$ is the traffic demand density from point s (on path $\mathbf{p}(s; \mathbf{r}, \theta)$) to $\mathbf{r}$, and $h(s) = h(s; \mathbf{x}, \mathbf{r})$ is "divergence rate of paths",

$$h(s) = \lim_{d\theta \to 0} \frac{p(s; \mathbf{r}, \theta + d\theta) - p(s; \mathbf{r}, \theta)}{d\theta}.$$

For example, with shortest paths $h(s) = s$. The proof is straightforward and essentially the same as given in [11, 12] for the resulting scalar packet flux with curvilinear paths.

2.2 Multi-path routing

Consider now the solution of the minmax problem (2) in the case that the traffic of each origin-destination pair can be arbitrarily split over all available routes (there are an infinite number of them). In analogy with a fixed network load balancing problem, our task can be formulated as an LP-type problem (over a set of decision variables with infinite cardinality),

$$\min_{\varphi(\mathbf{x}, \theta; \mathbf{r})} \alpha \tag{8}$$

such that

$$\alpha \geq \Phi(\mathbf{x}), \qquad \forall\, \mathbf{x}, \tag{9}$$

$$\nabla \cdot \mathbf{J}(\mathbf{x}, \mathbf{r}) = \lambda(\mathbf{x}, \mathbf{r}) - \delta(\mathbf{x} - \mathbf{r})\, \Lambda(\mathbf{r}), \qquad \forall\, \mathbf{x}, \mathbf{r}, \tag{10}$$

where $\Lambda(\mathbf{r})$ [1/s/m^2] denotes the density of total traffic destined to $\mathbf{r}$ (per unit area about $\mathbf{r}$),

$$\Lambda(\mathbf{r}) = \int_{\mathcal{A}} \lambda(\mathbf{x}, \mathbf{r})\, d^2\mathbf{x}.$$

The constraint (10) represents the requirement of flow continuity for each d-flow. Now we claim:

Proposition 1 *An optimal solution for the problem* (8)-(10) *can be obtained with single-path routing.*

PROOF. Let $\varphi^*(\mathbf{x}, \theta; \mathbf{r})$ be the optimal solution to the problem and let $\mathbf{J}^*(\mathbf{x}, \mathbf{r})$ and $\Phi^*(\mathbf{x})$ be the corresponding d-flow intensity and scalar flux. The claim follows by noting that the single-path forwarding defined by

$$\vartheta(\mathbf{x}, \mathbf{r}) = \arg \mathbf{J}^*(\mathbf{x}, \mathbf{r}), \tag{11}$$

yielding the following single-path angular d-flux

$$\varphi(\mathbf{x}, \theta; \mathbf{r}) = \delta(\theta - \arg \mathbf{J}^*(\mathbf{x}, \mathbf{r}))\, |\mathbf{J}^*(\mathbf{x}, \mathbf{r})| \tag{12}$$

satisfies the constraint (10), and that the associated scalar flux $\Phi(\mathbf{x})$ is everywhere less than or equal to $\Phi^*(\mathbf{x})$.

First, (10) is satisfied since

$$\mathbf{J}(\mathbf{x}, \mathbf{r}) = \int_0^{2\pi} \varphi(\mathbf{x}, \theta; \mathbf{r})\, \mathbf{e}_\theta\; d\theta = \mathbf{J}^*(\mathbf{x}, \mathbf{r}),$$

and $\mathbf{J}^*(\mathbf{x}, \mathbf{r})$ being a solution satisfying (10). Second, from the definition (12) we have

$$\int_0^{2\pi} \varphi(\mathbf{x}, \theta; \mathbf{r})\; d\theta = |\mathbf{J}^*(\mathbf{x}, \mathbf{r})|,$$

but from (3) it follows

$$\begin{aligned} |\mathbf{J}^*(\mathbf{x}, \mathbf{r})| &= \mathbf{J}^*(\mathbf{x}, \mathbf{r}) \cdot \mathbf{e}_{\mathbf{J}^*(\mathbf{x}, \mathbf{r})} \\ &= \int_0^{2\pi} \varphi^*(\mathbf{x}, \theta; \mathbf{r})\, \mathbf{e}_\theta \cdot \mathbf{e}_{\mathbf{J}^*(\mathbf{x}, \mathbf{r})}\; d\theta \\ &\leq \int_0^{2\pi} \varphi^*(\mathbf{x}, \theta; \mathbf{r})\; d\theta, \end{aligned} \tag{13}$$

where $\mathbf{e}_{\mathbf{J}^*(\mathbf{x}, \mathbf{r})}$ is the unit vector in direction of $\mathbf{J}^*(\mathbf{x}, \mathbf{r})$,

$$\mathbf{e}_{\mathbf{J}^*(\mathbf{x}, \mathbf{r})} = \mathbf{J}^*(\mathbf{x}, \mathbf{r}) / |\mathbf{J}^*(\mathbf{x}, \mathbf{r})|.$$

Thus we have established

$$\int_0^{2\pi} \varphi(\mathbf{x}, \theta; \mathbf{r})\; d\theta \leq \int_0^{2\pi} \varphi^*(\mathbf{x}, \theta; \mathbf{r})\; d\theta,$$

which implies $\Phi(\mathbf{x}) \leq \Phi^*(\mathbf{x})$ for all $\mathbf{x} \in \mathcal{A}$. But as $\varphi^*(\mathbf{x}, \theta; \mathbf{r})$ was assumed to be the optimal solution, it follows that the optimum can be obtained with the single-path routing (11). $\square$

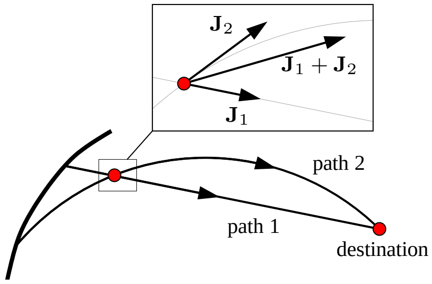

Figure 3: Transformation from a two-path routing to single-path routing according to (14)**.**

This means that for any multi-path routing the corresponding single-path routing, according to (11), yields at most equal scalar packet flux at every point $\mathbf{x}$. In particular, we have the following local result:

Remark 1 *If a multi-path solution at point* $\mathbf{x}$ *forwards packets with a given destination "genuinely" to several directions, then* (13) *in fact implies that the corresponding single-path routing, defined by* (11)*, yields a strictly lower scalar flux about* $\mathbf{x}$ *(cf., triangle inequality).*

2.3 Randomized path selection

Let us further elaborate the procedure of obtaining the single-path routes by considering a special case of multi-path routing obtained by randomly choosing a path for each packet destined to $\mathbf{r}$ from a countable set I of single-path solutions. In other words, each packet is marked to use a corresponding path from set I with a certain probability of α_i, $\sum_{i\in I}\alpha_i = 1$. Thus, the path chosen at random by the source is then followed to the destination without any further random selections. Let the $\mathbf{J}_i$ denote the corresponding d-flows when all the traffic destined to $\mathbf{r}$ is routed using path set i, $i \in I$. According to (11) the corresponding single-path routing decision is in the direction of the net d-flow, i.e.,

$$\vartheta(\mathbf{x},\mathbf{r}) = \arg \sum_i \alpha_i \mathbf{J}_i(\mathbf{x},\mathbf{r}). \tag{14}$$

This is illustrated in Fig. 3 for two alternative path sets. In the next section we will illustrate this by means of numerical examples and show that the single-path routing indeed gives a lower maximum scalar flux.

2.4 Uniqueness of the optimal solutions

Typically, an optimal solution for load balancing problem in a fixed network is such that the maximum load is obtained in several links. This suggests that the optimal solution in the present context of massively dense networks is also such that the maximum load, denoted by Φ_{opt}, is obtained in some area $\mathcal{A}^{(B)} \subset \mathcal{A}$. We further believe that $\mathcal{A}^{(B)}$ has strictly positive area, i.e., it is not a single point.

Definition 9 (bottleneck region) *Let* I *denote the set of all optimal solutions with regard to load balancing problem. Define the bottleneck region as*

$$\mathcal{A}^{(B)} = \{\mathbf{x} \in \mathcal{A} \,:\, \Phi_i(\mathbf{x}) = \Phi_{\text{opt}},\, \forall\, i \in I\}.$$

From the optimality it follows that the bottleneck region cannot be empty. To this end, consider an arbitrary finite subset of the optimal single-path solutions $\vartheta_i(\mathbf{x},\mathbf{r})$ yielding the scalar fluxes $\Phi_i(\mathbf{x})$, $i = 1,\ldots,n$. For the corresponding multi-path solution obtained by a randomized path selection with probabilities p_i, it holds that

$$\Phi(\mathbf{x}) = \sum_i p_i \cdot \Phi_i(\mathbf{x}).$$

In particular, if $\{\mathbf{x} \in \mathcal{A} \,:\, \Phi_i(\mathbf{x}) = \Phi_{\text{opt}},\, \forall\, i = 1,\ldots,n\} = \emptyset$, then choosing, e.g., $p_i = 1/n$ gives

$$\max_{\mathbf{x}} \left(\sum_i p_i \cdot \Phi_i(\mathbf{x})\right) < \Phi_{\text{opt}},$$

which is a contradiction and thus $\mathcal{A}^{(B)}$ is non-empty.

So for sure $\mathcal{A}^{(B)}$ contains at least one point. For now let us assume that the bottleneck region $\mathcal{A}^{(B)}$ is a compact set with strictly positive area.

Proposition 2 *Optimal paths are unambigious inside the bottleneck region.*

PROOF. Let $\vartheta_1(\mathbf{x},\mathbf{r})$ and $\vartheta_2(\mathbf{x},\mathbf{r})$ denote two optimal solutions. If

$$\vartheta_1(\mathbf{x},\mathbf{r}) \neq \vartheta_2(\mathbf{x},\mathbf{r}), \quad \text{for some } \mathbf{x} \in \mathcal{A}^{(B)},$$

then the corresponding single-path routing, obtained from the randomized multi-path routing using (14), would give us a better solution (cf., Remark 1). □

Proposition 3 *With strictly positive traffic demands,* $\lambda(\mathbf{x},\mathbf{r}) > 0$*, the optimal paths are bidirectional in the bottleneck region.*

PROOF. Without loss of generality, we can assume that $\mathbf{x} \in \mathcal{A}^{(B)}$. As the traffic demands are strictly positive there is some traffic flowing from $\mathbf{x}$ to $\mathbf{r}$, and vice versa. Let A denote the path from $\mathbf{x}$ to $\mathbf{r}$, and B the reverse path. Consider next a multi-path routing where some of the traffic on A has been moved to reverse path B, and similarly in the reverse direction. This clearly has no effect on the resulting scalar flux. However, if A is different from B (in the bottleneck region), then this multi-path solution could be improved by using the corresponding single-path solution, which leads to a contradiction. □

3. NUMERICAL EXAMPLES

Similarly as in the most of the previous work (see, e.g., [4, 9–12]), let us consider as an example the load balancing problem in unit disk with uniform traffic demands,

$$\lambda(\mathbf{x},\mathbf{r}) = \frac{\Lambda}{\pi^2}, \quad |\mathbf{x}|, |\mathbf{r}| \leq 1,$$

where Λ is the total packet flow. Due to the symmetry, the scalar flux in this system is a function of radius r only.

3.1 Shortest paths and radial-ring paths

Let us start by describing three elementary bidirectional single-path routes which also satisfy the destination based forwarding rule requirement given in Def. 7. All these path sets were studied already in [11] and [12] and here we just give brief review of results relevant to us. The most obvious set of such routes are the shortest path routes, i.e., straight line segments, for which the local routing rule at $\mathbf{x}$ towards destination at $\mathbf{r}$ is simply

$$\vartheta_{\text{sp}} = \arg(\mathbf{r} - \mathbf{x}).$$

The resulting scalar flux is given by integral, [11, 12]

$$\Phi_{\text{sp}}(r) = \frac{2(1-r^2)\cdot\Lambda}{\pi^2} \int_0^\pi \sqrt{1 - r^2\cos^2\phi}\, d\phi,$$

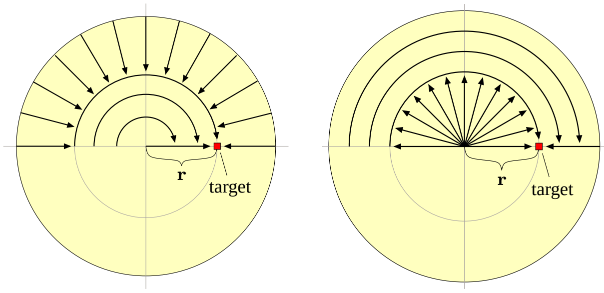

Figure 4: Local routing decisions $\vartheta(\mathbf{x}, \mathbf{r})$ for inner (left) and outer (right) radial-ring paths (in upper half plane).

which has its maximum at the origin,

$$\Phi_{\mathrm{sp}}(0) = \frac{2}{\pi}\Lambda \approx 0.637 \cdot \Lambda.$$

In Fig. 4 we have illustrated the single-path forwarding rules corresponding to the so-called inner and outer radial-ring paths studied in [11, 12]. These bidirectional single-path sets consist of one radial component and one ring component. With the inner radial-ring paths the order is chosen so that the ring component closer to origin is used, and for the outer radial-ring paths it is the opposite. These path sets are neither ideal, but their simple form facilitates the analysis and thus they serve as a good examples.

For unit disk, without loss of generality, one can assume that the destination is located on positive x-axis, $\mathbf{r} = (d, 0)$. The routing decision for radial-ring paths can be expressed conveniently in polar coordinates. Let (r, θ) denote the current location in upper half plane, for which we have the local routing rule for the inner radial-ring paths,

$$\vartheta_{\mathrm{in}} = \begin{cases} \theta - \pi, & \text{when } r > d, \\ \theta - \pi/2, & \text{when } r \le d \text{ and } \theta > 0, \\ 0, & \text{when } r < d \text{ and } \theta = 0, \end{cases}$$

and for the outer version,

$$\vartheta_{\mathrm{out}} = \begin{cases} \theta, & \text{when } r < d, \\ \theta - \pi/2, & \text{when } r \ge d \text{ and } \theta > 0, \\ \pi, & \text{when } r > d \text{ and } \theta = 0. \end{cases}$$

The corresponding scalar flux are

$$\Phi_{\mathrm{in}}(r) = \frac{(\pi+1)(r - r^3)}{\pi} \cdot \Lambda,$$

obtaining the maximum at $r = 1/\sqrt{3}$,

$$\Phi_{\mathrm{in}}(1/\sqrt{3}) \approx 0.507 \cdot \Lambda,$$

and for the outer version,

$$\Phi_{\mathrm{out}}(r) = \frac{(\pi-1)r^3 + r}{\pi} \cdot \Lambda,$$

obtaining the maximum flux at $r = 1$,

$$\Phi_{\mathrm{out}}(1) = \Lambda.$$

Note that, as mentioned earlier, for radial-ring paths several paths are combined together before reaching the destination and thus packets arriving at a certain angle may belong to different paths. For the shortest paths this is not the case.

Table 1: Results with the shortest paths, and 2- and 3- multi-path routes together with the respective single-path routes.

	proportions			max. flux	
	straight	outer	inner	multi-path	single-path
	1.00			0.637	(same)
1)	0.61	0.39		0.397	0.390
2)	0.503	0.376	0.121	0.376	0.344
3)	0.437	0.343	0.22	0.389	0.343

3.2 Randomized path selection

In [11,12] it was shown that by using a randomized path selection using two or more single-path routes from a given set of routes one can achieve considerably lower maximum scalar packet flux than with any of the single-path routes of the set alone. In particular, two combinations of the shortest paths and the radial-ring paths we considered:

i) shortest paths and outer radial-ring paths, and

ii) shortest paths and outer and inner radial-ring paths.

The optimized path selection probabilities were such that for the resulting scalar packet flux we have

$$\Phi_{\mathrm{mp1}}(r) = 0.61 \cdot \Phi_{\mathrm{sp}}(r) + 0.39 \cdot \Phi_{\mathrm{out}}(r),$$

and (subscript mp denotes "multi-path")

$$\Phi_{\mathrm{mp2}}(r) = 0.5027 \cdot \Phi_{\mathrm{sp}}(r) + 0.3763 \cdot \Phi_{\mathrm{out}}(r) + 0.121 \cdot \Phi_{\mathrm{in}}(r),$$

where the former yields a maximum flux of $0.397 \cdot \Lambda$, and the latter a maximum flux of $0.3763 \cdot \Lambda$, i.e., the flux corresponding to the outer radial-ring paths at the boundary. The numerical results are given in Table 1, where rows indicated with 1) and 2) correspond to the optimal weights for randomized path selection with the given two and three path sets, respectively, and column "multi-path" contains the corresponding maximum scalar fluxes.

However, according to Proposition 1, multi-path routes $\mathrm{mp1}$ and $\mathrm{mp2}$ cannot be an optimal solution to the load balancing problem, and, in particular, the corresponding single-path routes, denoted by $\mathrm{sp1}$ and $\mathrm{sp2}$, obtained using (14) yield a lower maximum scalar packet flux. This maximum scalar flux can be computed numerically and the corresponding results are given in column "single-path" in Table 1. We note that in both cases combining the multi-path traffic flows to single-path improves the situation considerably, as expected.

3.3 Further Optimization

Instead of using the proportions optimal with respect to the randomized multi-path routing, one can also treat the route selection probabilities α_i as free optimization parameters for the resulting single-path routing. As an example, let us consider combinations of the same basic routes consisting of two and three path sets. It turns out, that in this case, the optimal proportions for the two path sets (straight and outer) remain the same, as given in row 1) in Table 1. However, for the three path sets (straight, outer and inner) the optimal proportions are different and are given in row 3). In particular, with these optimized weights the corresponding single-path route set, denoted by $\mathrm{sp3}$, yields a maximum scalar packet flux of $0.343 \cdot \Lambda$, which is, to best of our knowledge, considerably less than what is obtained with any previously proposed set of paths. For the reference, the circular paths introduced in [11] yield a maximal flux of $0.424 \cdot \Lambda$, and in [12] a modified version of this path set, after numerical optimization, gives a maximal flux of $0.384 \cdot \Lambda$.

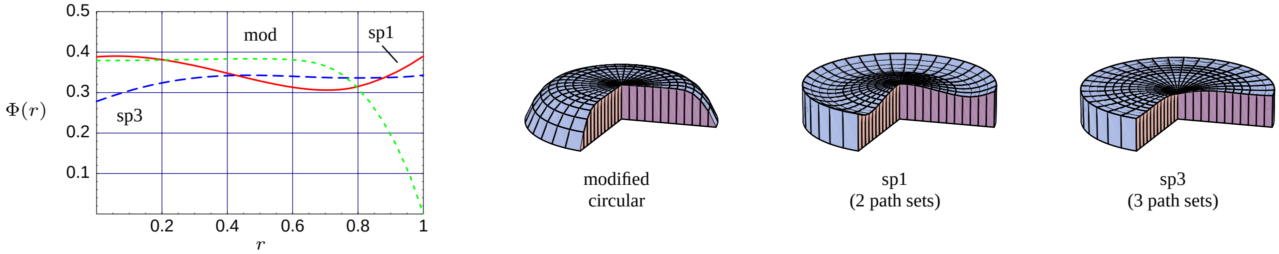

Figure 5: Resulting scalar flux as a function of distance r from the origin for modified circular paths (see [12]), and the optimal single-path routes $\mathrm{sp1}$ and $\mathrm{sp3}$ (rows 1) and 3) in Table 1). Three dimensional plots illustrate the same situation.

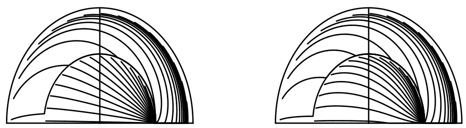

Figure 6: Single-path routes towards point $(0.6, 0)$ obtained by combining multi-path routes $\mathrm{mp1}$ and $\mathrm{mp3}$ to corresponding single-path routes $\mathrm{sp1}$ and $\mathrm{sp3}$ (rows 1) and 3) in Table 1).

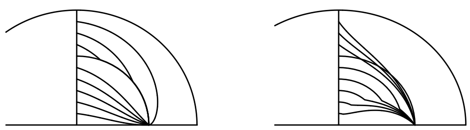

Figure 7: Single-path routes $\mathrm{sp3}$ from the positive x-axis to $(0.6, 0)$ (left fig.), and the reverse paths from $(0.6, 0)$ back to positive x-axis (right fig.). Paths are clearly not bidirectional.

The resulting scalar fluxes for the single-path routes sp1 and sp3 are illustrated in Fig. 5. Interestingly, with sp3 the scalar packet flux at the center of the area is clearly lower than the maximum. This suggests that there is still some room for improvement, e.g., by choosing a different set of base routes.

Fig. 6 illustrates the single-path routes sp1 and sp3 for destination point $\mathbf{r} = (0.6, 0)$ (the lower half plane is symmetric). From the figure it can be seen that the ring with radius 0.6 has a specific role and acts as a "highway" towards the destination. This is due to the fact that both radial-ring paths guide most of the traffic going to $(0.6, 0)$ to this ring yielding a singularity in the corresponding d-flow (i.e., a delta function). This singularity is then also present in the resulting single-path routes. Intuitively, from the figure one can see that neither of these single-path routes can be optimal, e.g., paths just above point $(1 - \epsilon, 0)$ are perpendicular to the "correct" direction. The resulting paths deviate from the intuitive ones even more when $d \to 0$ or when $d \to 1$.

Finally we note that the obtained single-path routes are not generally bidirectional. This can be seen, e.g., from Fig. 7 which illustrates the single-path routes $\mathrm{sp3}$. The left graph depicts paths from the positive x-axis to $(0.6, 0)$ and the graph on right depicts the reverse paths from $(0.6, 0)$ back to the positive x-axis. Clearly the paths are different and thus the single-path routes are not bidirectional. This along with the fact that some of the reverse paths cross each other two times suggests that the paths are not optimal and the result can further be improved.

3.4 Routes according to heat conduction

The circular paths studied in [11, 12] can be related to the optical paths with a certain index of refraction. An alternative set of routes can be obtained by considering heat conduction in a given homogeneous area. The temperature field $\phi(\mathbf{x})$ in stationary state is governed by the equation (with appropriately chosen constants)

$$\nabla^2 \phi(\mathbf{x}) = -q(\mathbf{x}),$$

where $q(\mathbf{x})$ is the heat source density. In particular, considering a heat sink at point $\mathbf{r}$ and, e.g., uniformly distributed heat sources in given area yields a vector field corresponding to the heat flowing towards the sink at $\mathbf{r}$. These heat flows can be interpreted as paths to $\mathbf{r}$ fulfilling the single-path routing requirement given in Def. 7 (i.e., destination based forwarding).

Heat flow in a unit disk can be solved by the well-known methods of complex analysis, see, e.g., [13]. Let the position of a point (x, y) on the disk be represented by the complex number $z = x + iy$. Let $w(z) = \phi(z) + i\psi(z)$ be the complex potential function such that the curves $\phi(z) = \text{const.}$ represent isotherms and curves $\psi(z) = \text{const.}$ represent the flow lines of heat flow field. The potential function $w(z)$ of a unit flow between a source at z_1 and a sink at z_2 is given by

$$w(z) = \varphi_+(z, z_2) + \varphi_-(z, z_2^*) - \varphi_+(z, z_1) - \varphi_-(z, z_1^*),$$

where $(\cdot)^*$ denotes complex conjugation and

$$\varphi_\pm(z, z_1) = \log\left(\frac{1 - z}{1 + z} \mp \frac{1 - z_1}{1 + z_1}\right).$$

Potential $\phi(z)$ is obtained as the real part of $w(z)$. Explicitly, denoting the the source and destination location vectors corresponding to z_1 and z_2 by $\mathbf{s}$ and $\mathbf{r}$, we get (up to a constant that depends on $\mathbf{s}$ and $\mathbf{r}$ but not on $\mathbf{x}$)

$$\phi(\mathbf{x}, \mathbf{s}, \mathbf{r}) = \frac{1}{4\pi} \log\left(\frac{(x - r_x)^2 + (y - r_y)^2}{(x - s_x)^2 + (y - s_y)^2} \cdot \frac{1 - 2xr_x - 2yr_y + (x^2 + y^2)(r_x^2 + r_y^2)}{1 - 2xs_x - 2ys_y + (x^2 + y^2)(s_x^2 + s_y^2)}\right).$$

The d-flow intensity $\mathbf{J}(\mathbf{x}, \mathbf{r})$ directed to destination $\mathbf{r}$ for uniformly distributed sources and sinks with total traffic Λ is obtained by integrating with respect to $\mathbf{s}$ over the whole disk,

$$\mathbf{J}(\mathbf{x}, \mathbf{r}) = -\frac{\Lambda}{\pi^2} \int \nabla_{\mathbf{x}} \phi(\mathbf{x}, \mathbf{s}, \mathbf{r})\, d^2\mathbf{s}. \tag{15}$$

Note that here, similarly as in the case considered in 3.2, through every point $\mathbf{x}$ there is a multiplicity of paths (corresponding to different source points $\mathbf{s}$) going to $\mathbf{r}$. Again, by Proposition (1), a lower scalar flux is obtained by replacing the routes to $\mathbf{r}$ with the flow lines of the flow field (15). The resulting scalar packet flux, $\Phi(\mathbf{x}) = \int |\mathbf{J}(\mathbf{x}, \mathbf{r})| d^2\mathbf{r}$, is depicted in Fig. 8 together with the scalar flux corresponding to circular paths. Interestingly, it turns out that the maximum scalar flux obtained in the center of the unit

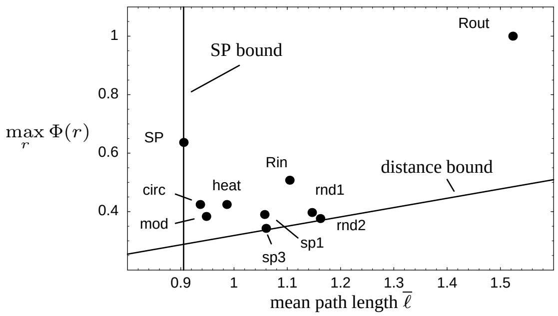

Figure 9: Trade-off between maximum scalar flux (y-axis) and the mean path length (x-axis) for different path sets together with two lower bounds (see [11, 12]).

disk is exactly the same, 0.424, for these two path sets. However, elsewhere the scalar packet fluxes are different. The paths according to heat flow push more traffic to the outskirts of the area, while in the middle there is less traffic. Computing the average scalar packet flux, which is directly proportional to the mean path length, reveals that the circular paths are indeed better as they yield a slightly lower average scalar packet flux, i.e., the average path length is slightly smaller. Both fall short of the result obtained in 3.3 with the single-path routing derived from 3-path routing.

3.5 Discussion

The shortest paths yield by definition the shortest possible mean path length, or equivalently, the smallest possible average scalar packet flux. In order to lower the maximum scalar packet flux one needs to bend some paths away from the congested area, which increases the mean path length and thus also the average scalar flux. In other words, there is a trade-off between the achievable minimum maximum scalar flux and the average scalar flux. In particular, from [11, 12],

$$\max_{\mathbf{r}} \Phi_{\mathcal{P}}(\mathbf{r}) \geq \frac{\Lambda \cdot \bar{\ell}(\mathcal{P})}{A}, \qquad (16)$$

where $\bar{\ell}(\mathcal{P})$ denotes the mean path length with path set $\mathcal{P}$. This is illustrated in Fig. 9. Especially, we observe that the new solution sp3 is very close to lower bound (16). In fact, all path sets located on the distance bound (16) have flat scalar packet flux distribution.

4. CONCLUSIONS

In this paper we have considered the load balancing problem in massively dense (wireless) multi-hop networks, i.e., by choosing the routes appropriately our aim is to decrease the maximum nodal forwarding load in the network represented by the so-called scalar flux. In particular, we have focused on comparing the multi-path routes to the single-path routes, and showed that in this case the optimal load balancing can always be obtained by a single-path routes between each source-destination pair, which is the main contribution of this paper. This is in strike contrast with the traditional fixed networks, where the use of multiple paths often yields a better load balancing and higher throughput in the network. Moreover, we have shown that in the bottleneck area the optimal paths are bidirectional meaning that the same path is traversed in both directions.

By using the single-path routes, we were able to find new solutions for the example case of a multi-hop network in a unit disk with uniform traffic demands. The new routes, to best of our knowledge, outperform the previously known single- and multi-path solutions [11, 12]. In particular, the best optimized single-path solution yields a maximum packet flux of $0.343 \cdot \Lambda$ corresponding to about 46% improvement when compared to the shortest path routes, which tend to concentrate unnecessarily much traffic in the center of the network (in the uniform case).

Acknowledgements

For the part of E. Hyytiä, this work has been performed in the Telecommunications Research Center Vienna (ftw.) in the framework of the Austrian Kplus Competence Centre programme. For the part of J. Virtamo, this work was performed in the project Fancy funded by the Academy of Finland (grant n:o 210275).

5. REFERENCES

[1] Dimitri Bertsekas and Robert Gallager, *Data Networks*, Prentice-Hall, 2nd edition, 1992.

[2] S. Toumpis and L. Tassiulas, “Optimal deployment of large wireless sensor networks,” *IEEE Trans. Information Theory*, vol. 52, no. 7, pp. 2935–2953, 2006.

[3] M. Kalantari and M. Shayman, “Routing in wireless ad hoc networks by analogy to electrostatic theory,” in *IEEE International Conference on Communications (ICC’04)*, Paris, France, June 2004, pp. 4028–4033.

[4] Philippe Jacquet, “Geometry of information propagation in massively dense ad hoc networks,” in *ACM MobiHoc*, Roppongi Hills, Japan, 2004, pp. 157–162.

[5] S. Toumpis, “Optimal design and operation of massively dense wireless networks,” in *ACM Inter-Perf’06*, Oct. 2006.

[6] Roberto Catanuto and Giacomo Morabito, “Optimal routing in dense wireless multihop networks as a geometrical optics solution to the problem of variations,” in *IEEE International Conference on Communications (ICC’06)*, Istanbul, Turkey, June 2006.

[7] Roberto Catanuto, Stavros Toumpis, and Giacomo Morabito, “Opti{c,m}al: Optical/optimal routing in massively dense wireless networks,” in *Proc. of IEEE Infocom ’07*, Anchorage, Alaska, May 2007.

[8] Ness Shroff and Sungoh Kwon, “Paradox of shortest path routing for large multi-hop wireless networks,” in *Proc. of IEEE Infocom ’07*, Anchorage, Alaska, May 2007.

[9] Peter P. Pham and Sylvie Perreau, “Performance analysis of reactive shortest path and multi-path routing mechanism with load balance,” in *Proc. of IEEE Infocom ’03*, San Francisco, USA, March-April 2003, vol. 1, pp. 251–259.

[10] Yashar Ganjali and Abtin Keshavarzian, “Load balancing in ad hoc networks: Single-path routing vs. multi-path routing,” in *Proc. of IEEE Infocom ’04*, Hong Kong, Mar. 2004.

[11] Esa Hyytiä and Jorma Virtamo, “On load balancing in a dense wireless multihop network,” in *NGI 2006, 2nd Conference on Next Generation Internet Design and Engineering*, València, Spain, Apr. 2006.

[12] Esa Hyytiä and Jorma Virtamo, “On traffic load distribution and load balancing in dense wireless multihop networks,” *EURASIP Journal on Wireless Communications and Networking*, June 2007, Special Issue on Novel Techniques for Analysis & Design of Cross-Layer Optimized Wireless Sensor Networks.

[13] I. Sneddon, *Elements of partial differential equations*, McGraw-Hill Book Company, 1957.

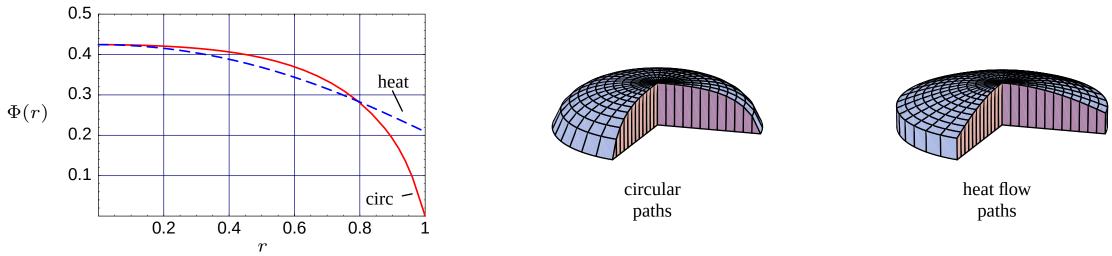

Figure 8: Resulting scalar flux as a function of distance r from the origin for the circular paths (see [11, 12]) and for the paths according to heat flow. Three dimensional plots illustrate the same situation.

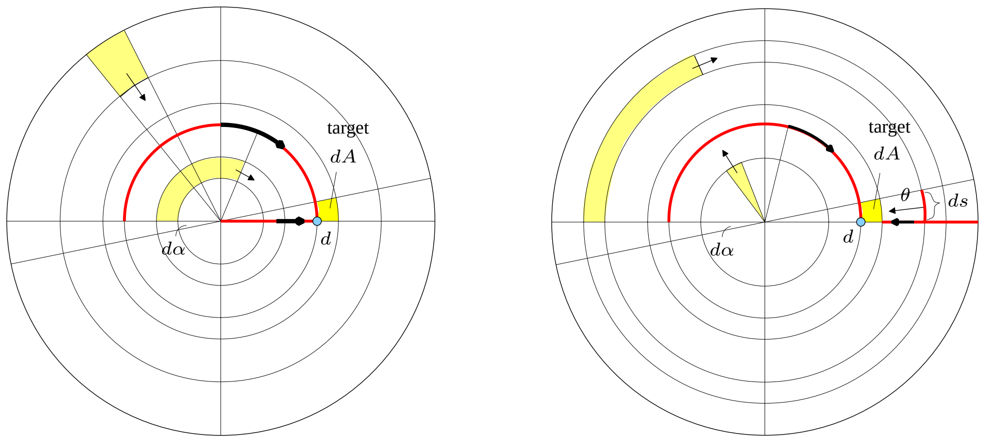

Figure 10: Derivation of d-flow intensity for radial-ring paths: inner (left) and outer (right).

APPENDIX

A. SOME D-FLOW INTENSITIES

Let us explicitly write down the d-flow intensities for SP and radial-ring paths in unit disk with uniform traffic demand density, $\lambda(\mathbf{x}, \mathbf{r}) = \Lambda/\pi^2$. Note that for single-path routes the angular d-flux consists of delta-function. Without loss of generality we can assume that the destination is on the positive x-axis at point $\mathbf{r}=(d, 0)$.

For radial-ring paths, consider the upper half plane and denote the source point $\mathbf{x}$ with (r, α) in polar coordinates. Starting from Fig. 10, it is straightforward to derive d-flow intensities, which are given in Table 2. (Due to lack of space the details are omitted.)

For the shortest paths it is more convenient to use cartesian coordinates (x, y) for point $\mathbf{x}$. Let s_0 denote the distance between $\mathbf{r}$ and $\mathbf{x}$, $z = |\mathbf{x} - \mathbf{r}|$, and s_1 the distance to the boundary from $\mathbf{r}$ in the direction of source point $\mathbf{x}$. For these we have,

$$z = \sqrt{d^2 - 2\,d\,x + x^2 + y^2},$$

$$a = \frac{d^2 - d\,x + \sqrt{d^2 - 2\,d\,x + x^2 + (1-d^2)y^2}}{\sqrt{d^2 - 2\,d\,x + x^2 + y^2}}.$$

Then, (7) immediately yields

$$\mathbf{J}_{\rm sp}(x, y; d) = \frac{\Lambda}{\pi^2} \cdot \frac{a^2 - z^2}{2z}\,\mathbf{e}_{\mathbf{r}-\mathbf{x}}, \qquad \text{when } z > 0,$$

which can be further simplified yielding the expression in Table 2.

B. FORMULATION IN FIXED NETWORK

The load balancing problem in traditional fixed network can be formulated as an LP-problem. Let d_{ij}, $i \neq j$, denote the traffic demand from i to j, and d_{jj} the total traffic offered to i, $\sum_{i\neq j} d_{ij} = -d_{jj}$. Let $\lambda_{ij}^{(k)}$ denote the traffic on link $i \to j$ having the final destination at k. Then the load balancing problem can be stated as,

$$\min \alpha$$

such that

$$\sum_k \lambda_{ij}^{(k)} \leq \alpha \qquad \forall\, i, j,$$

$$\sum_{i \in L(j)} \lambda_{ji}^{(k)} - \lambda_{ij}^{(k)} = d_{jk} \qquad \forall\, j, k,$$

where $L(j)$ denotes the set of nodes connected to node j.

Table 2: d-flow in unit disk for SP and radial-ring paths.

Inner radial-ring paths:

$$\mathbf{J}_{\rm in}(r, \alpha; d) = \frac{\Lambda}{\pi^2} \begin{cases} (1-r^2)/(2r)\,\mathbf{e}_{\alpha-\pi}, & r>d, \\ (\pi-\alpha)(r+(1-d^2)\cdot\delta(r-d))\,\mathbf{e}_{\alpha-\pi/2}, & r\leq d,\ \alpha>0, \\ \pi r \cdot \delta(\alpha)\,\mathbf{e}_0, & r<d,\ \alpha=0, \\ 0, & r=d,\ \alpha=0. \end{cases}$$

Outer radial-ring paths:

$$\mathbf{J}_{\rm out}(r, \alpha; d) = \frac{\Lambda}{\pi^2} \begin{cases} (r/2)\,\mathbf{e}_{\alpha}, & r<d, \\ (\pi-\alpha)r(1+d\cdot\delta(r-d))\,\mathbf{e}_{\alpha-\pi/2}, & r\geq d,\ \alpha>0, \\ \pi(1-r^2)/r\cdot\delta(\alpha)\,\mathbf{e}_{\pi}, & r>d,\ \alpha=0, \\ 0, & r=d,\ \alpha=0. \end{cases}$$

Shortest paths:

$$\mathbf{J}_{\rm sp}(x, y; d) = \frac{\Lambda}{2\pi^2}\left[\frac{\left(d(d-x)+\sqrt{(d-x)^2+(1-d^2)y^2}\right)^2}{((d-x)^2+y^2)^2} - 1\right](d-x, -y).$$