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P2P VoD Systems: Modelling and Performance

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Contents

- Peer-to-peer systems
- File sharing: fluid model
- File sharing: steady-state analysis
- File sharing: conclusions
- Video-on-demand: fluid model
- Video-on-demand: steady-state analysis
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- Video-on-demand: conclusions

Fundamental principle

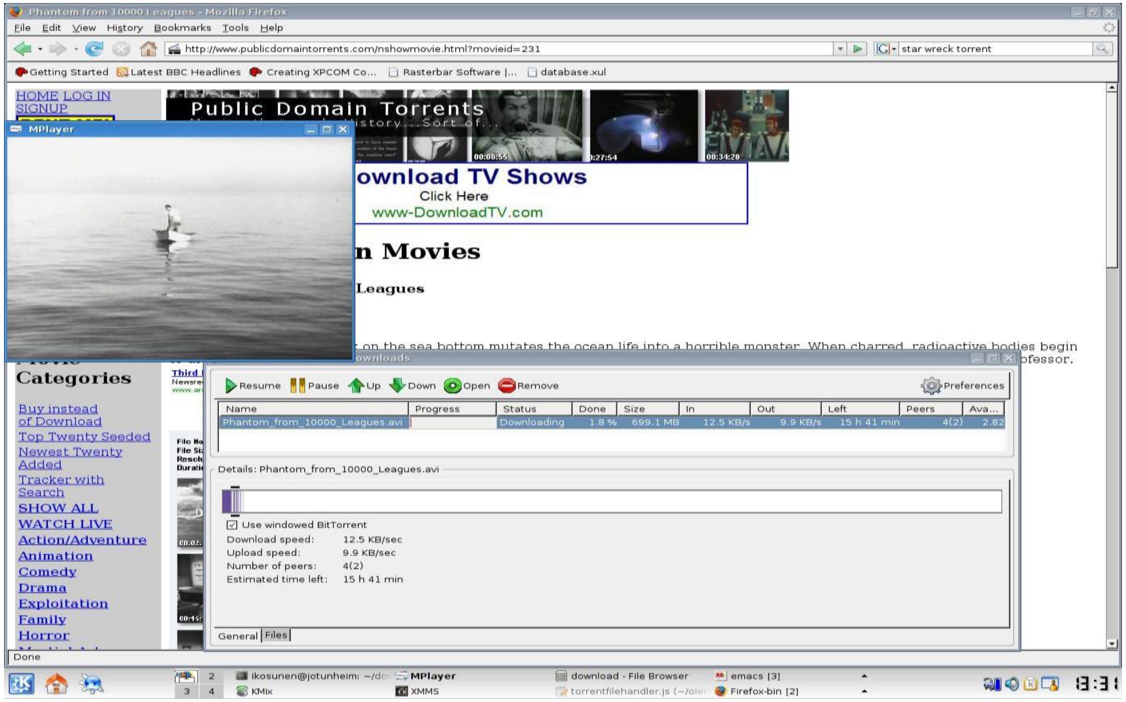
- Client/Server (CS) paradigm
  - **Clients** download content from **servers**
  - Clear distinction between the two roles
  - Service capacity remains the same, while load increases
  - When too many clients, transfer times explode
  - Offered load bounded by this stability limit (for sure!)

- Peer-to-peer (P2P) systems
  - **Peers** download pieces of content from other peers/seeds and **simultaneously** upload downloaded pieces to other peers
  - Blurring of roles: peers not only act as clients (when downloading) but also serve other peers (when uploading)
  - Service capacity scales with the offered load
  - No stability limit (for sure?)

Applications

- P2P used commonly for file sharing (e.g. BitTorrent) and live streaming

- P2P video-on-demand (VoD):
  - Alternative to client-server approaches (YouTube)?
  - Under what conditions?



Quality of Service

- P2P file sharing
  - Retrieve the whole file as soon as possible
  - Retrieve pieces in any order
  - Minimize the file transfer time

- P2P streaming
  - Retrieve pieces at least at playback rate and in almost sequential order
  - Minimize the startup delay (needed to fill the playout buffer)

- P2P video-on-demand
  - Retrieve the whole file
  - Retrieve pieces at least at playback rate and in almost sequential order
  - Minimize the startup delay (needed to fill the playout buffer)

Why performance modelling?

- Scalability
  - Is the system really scalable?
- Stability
  - If not, where is the stability limit for the load?
- Performance
  - When stable, is the performance sufficient?

Modelling aspects

- Dynamic population model
  - describing the evolution of the peer population in the P2P system
- Peer arrival process
  - steady arrival rate, smoothly attenuating arrival rate, or flash crowd?
- Efficiency of resource sharing
  - utilization of a peer’s upload capacity
  - effect of the piece/peer selection policy
  - number of parallel connections
- Selfishness / altruism
  - part of peers are free-riders that do not want to share upload capacity
- Download and upload rates
  - homogeneous or heterogeneous peer population?
- Number of permanent seeds
  - correspond to servers in the client-server architecture

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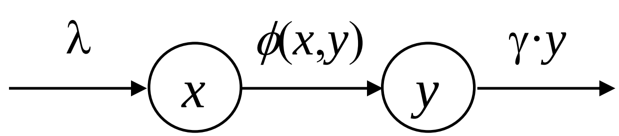
Model for P2P file sharing

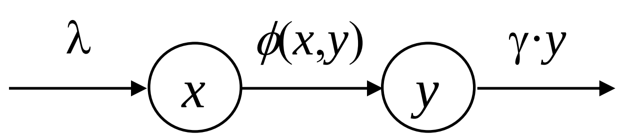
- Life span of a peer consists of two sequential phases:
  - file transfer phase, during which the peers are called leechers
  - sharing phase, during which the peers are called seeds
- Altruistic peers have a longer sharing phase than selfish peers

- Model by Qiu and Srikant (2004):
  - deterministic fluid model (= system of differential equations)
  - describing the system dynamics related to sharing of a single file
  - $x(t)$ = (average) number of leechers at time t
  - $y(t)$ = (average) number of non-permanent seeds at time t



Assumptions

- Steady arrival process described by
  - arrival rate λ to transfer phase (arrivals per time unit)
- Efficiency described by
  - upload utilization ratio η (belonging to (0,1])
- Selfishness described by
  - departure rate γ from service phase (departures per time unit)
- Homogeneous peer population with
  - download rate c (file transfers per time unit) and
  - upload rate μ (file transfers per time unit)
- No permanent seeds



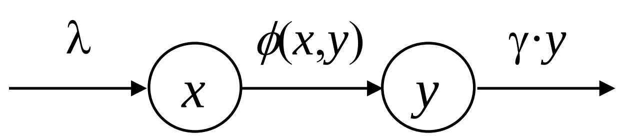
Fluid model

- Switched nonlinear system:

$$\begin{cases} x'(t) = \lambda - \phi(t), \\ y'(t) = \phi(t) - \gamma y(t), \end{cases} \tag{1}$$

- Aggregate service rate:

$$\phi(t) = \min\{cx(t), \mu(\eta x(t) + y(t))\}. \tag{2}$$



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Steady-state analysis

- Solve the equilibrium of the system by setting $x'(t) = y'(t) = 0$ in (1):

$$\begin{cases} \bar{\phi} = \lambda, \\ \bar{y} = \dfrac{\lambda}{\gamma}, \end{cases} \tag{3}$$

- Two cases considered separately:
  - download-constrained system in equilibrium
  - upload-constrained system in equilibrium

- Parameter space divided nicely in two complementary parts each of which has a unique equilibrium solution
  - that are even globally stable by Qiu and Sang (2008)

Download-constrained system

If

$$c\bar{x} \le \mu(\eta\bar{x} + \bar{y}), \tag{4}$$

then the system is *download-constrained*, and

$$c\bar{x} = \bar{\phi} = \lambda, \tag{5}$$

implying that

$$\begin{cases} \bar{x}_d = \dfrac{\lambda}{c}, \\ \bar{y}_d = \dfrac{\lambda}{\gamma}. \end{cases} \tag{6}$$

The resulting condition for a download-constrained system:

$$\frac{1}{\mu} \le \frac{\eta}{c} + \frac{1}{\gamma}. \tag{7}$$

Upload-constrained system

If

$$c\bar{x} > \mu(\eta\bar{x} + \bar{y}), \tag{8}$$

then the system is *upload-constrained*, and

$$\mu(\eta\bar{x} + \bar{y}) = \bar{\phi} = \lambda, \tag{9}$$

implying that

$$\begin{cases} \bar{x}_u = \dfrac{\lambda}{\eta}\left(\dfrac{1}{\mu} - \dfrac{1}{\gamma}\right), \\ \bar{y}_u = \dfrac{\lambda}{\gamma}. \end{cases} \tag{10}$$

The resulting condition for a upload-constrained system:

$$\frac{1}{\mu} > \frac{\eta}{c} + \frac{1}{\gamma}. \tag{11}$$

Deterministic model vs. stochastic simulations

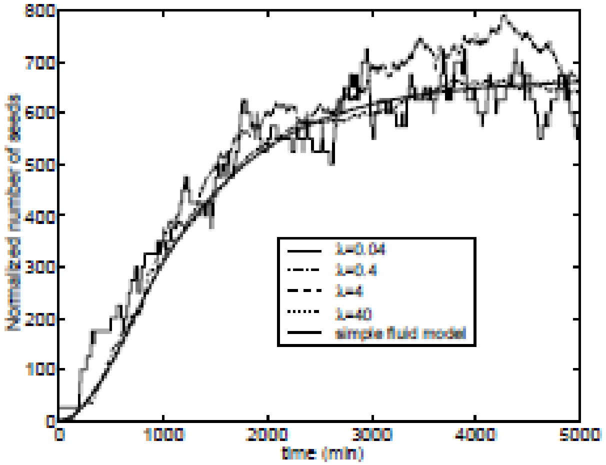

Figure 1: Experiment 1 : The evolution of the number of seeds as a function of time

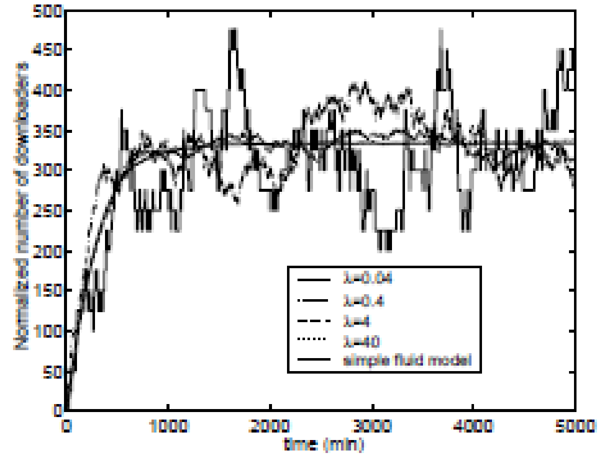

Figure 2: Experiment 1 : The evolution of the number of downloaders as a function of time

Source: Qiu and Srikant (2004)

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Conclusions from the P2P file sharing model

- Scalability
  - System scalable in the whole parameter space by (6) and (10), in particular for any $\eta > 0$
- Stability
  - Consequently, system stable for any $\lambda > 0$
- Performance
  - By Little's formula, the mean file transfer time is

$$T = \frac{\bar{x}}{\lambda} = \max\{\frac{1}{c}, \frac{1}{\eta}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)\} \leq \max\{\frac{1}{c}, \frac{1}{\eta\mu}\} \approx \max\{\frac{1}{c}, \frac{1}{\mu}\}. \qquad (12)$$

  - Thus, no real problems in performance if reasonable download and upload rates with respect to the mean file size
  - The last approximation justified for the file sharing application (mainly due to the free retrieving order of pieces)

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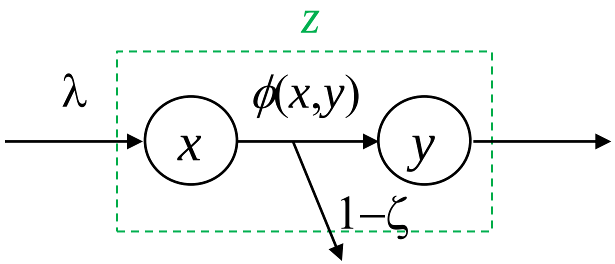
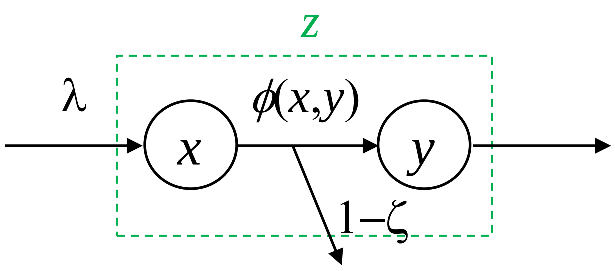
Model for P2P VoD

- Life span of a peer consists of two overlapping phases:
  - file transfer phase, during which the peers are called leechers
  - watching phase, starting immediately after the initial buffering delay
- Altruistic peers become seeds after the file transfer phase if the watching phase still continues
- Model by Aalto et al. (2010):
  - deterministic fluid model (= system of differential equations)
  - describing the system dynamics related to sharing of a single video file
  - $x(t)$ = (average) number of leechers at time t
  - $y(t)$ = (average) number of non-permanent seeds at time t



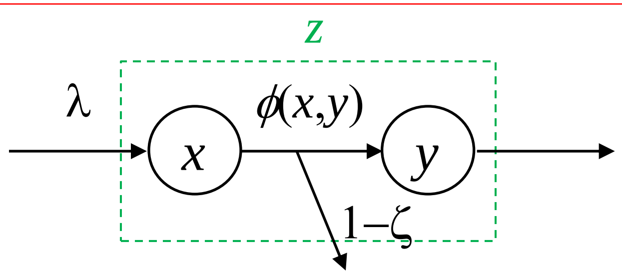
Assumptions (1)

- Steady arrival process described by
  - arrival rate λ (arrivals per time unit)
- Efficiency described by
  - upload utilization ratio η (belonging to (0,1])
- Altruism described by
  - probability ζ (for a peer to become a seed)
- Homogeneous peer population with
  - download rate c (file transfers per time unit) and
  - upload rate μ (file transfers per time unit)
- Number of permanent seeds = k (belonging to {0,1,2,…})



Assumptions (2)

- Startup delay negligible (if video sufficiently long)
  - Thus, the transfer phase and the playback phase start essentially at the same time
- Video watched at (fixed) playback rate
  - Total watching time denoted by z
  - Natural requirement: $z > 1/c$ (since transfer rate always bounded by c)
- Playback quality problems if the transfer phase takes longer than z
  - In this case, the playback phase ends as soon as the transfer is completed
- Selfish peers stay in the system until the end of the transfer phase while altruist peers stay until the end of the playback phase
  - but no longer, which is a worst case scenario



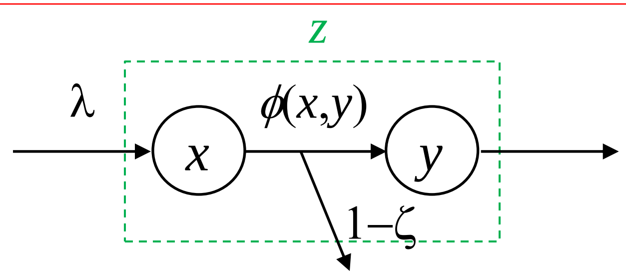
Fluid model

- Switched nonlinear system:

$$\begin{cases} x'(t) = \lambda - \phi(t), \\ y'(t) = \zeta\phi(t) - \dfrac{y(t)}{z - x(t)/\lambda}, \end{cases} \qquad (13)$$

- Aggregate service rate:

$$\phi(t) = \min\{cx(t), \mu(\eta x(t) + y(t) + k)\}. \qquad (14)$$



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Steady-state analysis

- Solve the equilibrium of the system by setting $x'(t) = y'(t) = 0$ in (13):

$$\begin{cases} \bar{\phi} = \lambda, \\ \bar{y} = \zeta\lambda(z - \bar{x}/\lambda), \end{cases} \tag{15}$$

- Two cases considered separately:
  - download-constrained system in equilibrium
  - upload-constrained system in equilibrium

- Multiple solutions found
- Local stability analysis used to rule out some of them

Download-constrained system

If

$$c\bar{x} \leq \mu(\eta\bar{x} + \bar{y} + k), \tag{16}$$

then the system is *download-constrained*, and

$$c\bar{x} = \bar{\phi} = \lambda, \tag{17}$$

implying that

$$\begin{cases} \bar{x}_d = \dfrac{\lambda}{c}, \\ \bar{y}_d = \zeta\lambda\left(z - \dfrac{1}{c}\right). \end{cases} \tag{18}$$

The resulting condition for a download-constrained system:

$$\frac{1}{\mu} \leq \frac{\eta}{c} + \zeta\lambda\left(z - \frac{1}{c}\right) + \frac{k}{\lambda}. \tag{19}$$

Upload-constrained system (1)

If

$$c\bar{x} > \mu(\eta\bar{x} + \bar{y} + k), \tag{20}$$

then the system is *upload-constrained*, and

$$\bar{\phi} = \mu(\eta\bar{x} + \bar{y} + k), \tag{21}$$

implying that

$$\begin{cases} \bar{x}_u = \dfrac{\lambda}{\eta - \zeta}\left(\dfrac{1}{\mu} - \zeta z - \dfrac{k}{\lambda}\right), \\ \bar{y}_u = \dfrac{\zeta\lambda}{\eta - \zeta}\left(-\dfrac{1}{\mu} + \eta z + \dfrac{k}{\lambda}\right). \end{cases} \tag{22}$$

The resulting conditions for a upload-constrained system: $\eta \neq \zeta$ and

$$\begin{cases} \dfrac{1}{\mu} > \dfrac{\eta}{c} + \zeta\left(z - \dfrac{1}{c}\right) + \dfrac{k}{\lambda}, & \text{if } \eta > \zeta, \\ \dfrac{1}{\mu} < \dfrac{\eta}{c} + \zeta\left(z - \dfrac{1}{c}\right) + \dfrac{k}{\lambda}, & \text{if } \eta < \zeta. \end{cases} \tag{23}$$

Upload-constrained system (2)

Additionally, for the solution to be meaningful, we require that $\bar{x}_{\mathrm{u}} > 0$ and $\bar{y}_{\mathrm{u}} > 0$. The former one follows from (23), but the latter one leads to the following additional constraints:

$$\begin{cases} \zeta < \dfrac{1}{z}\left(\dfrac{1}{\mu} - \dfrac{k}{\lambda}\right) < \eta, & \text{if } \eta > \zeta, \\ \eta < \dfrac{1}{z}\left(\dfrac{1}{\mu} - \dfrac{k}{\lambda}\right) < \zeta, & \text{if } \eta < \zeta, \end{cases} \tag{24}$$

which implies that $0 < \frac{1}{\mu} - \frac{k}{\lambda} < z$ is a necessary condition for the existence of a non-negative upload constrained solution.

Summary of the steady-state analysis (1)

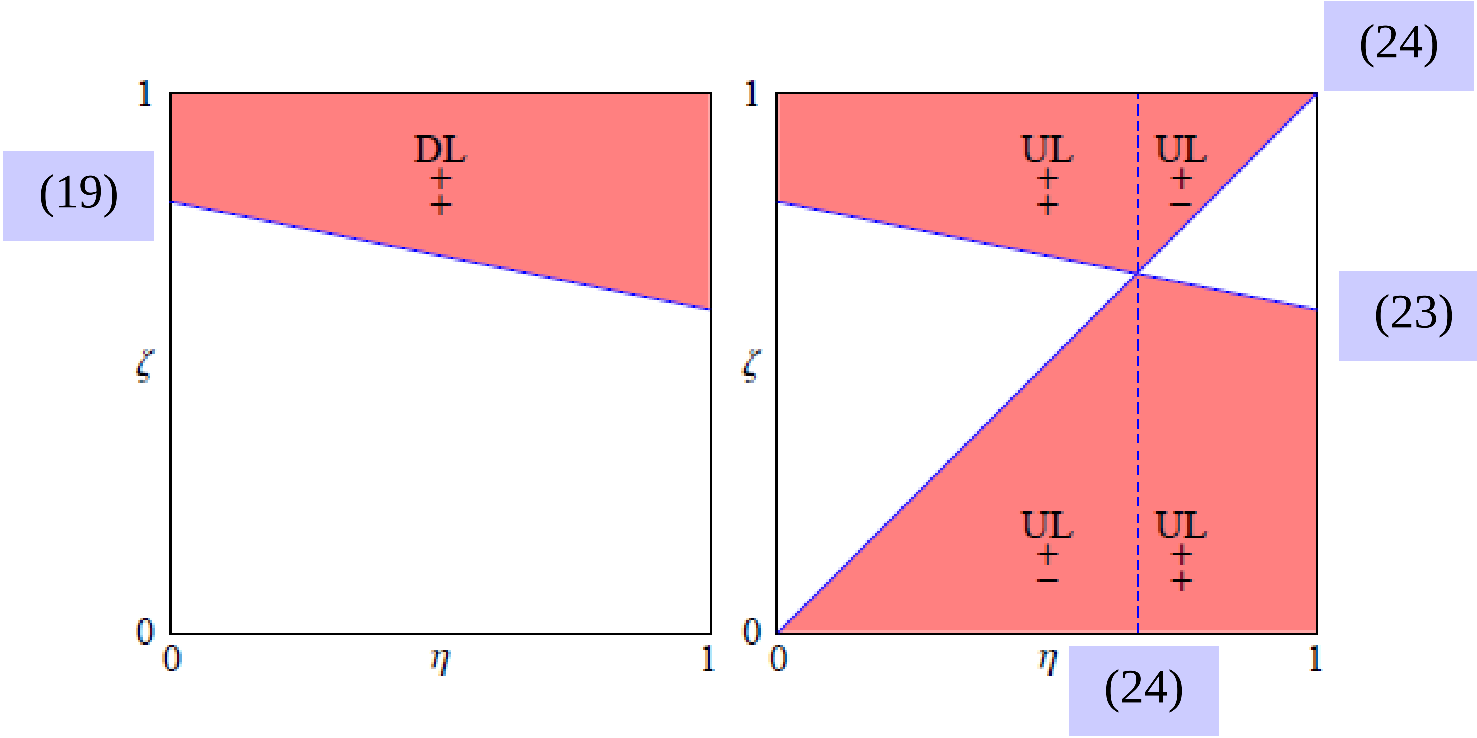

Fig. 1. *Left panel*: Download-constrained solution with $+/+$ expressing that $\bar{x}_{\mathrm{d}} > 0$ and $\bar{y}_{\mathrm{d}} > 0$ in this area. *Right panel*: Upload-constrained solution with $+/+$ $[+/-]$ expressing that $\bar{x}_{\mathrm{u}} > 0$ and $\bar{y}_{\mathrm{u}} > 0$ $[\bar{y}_{\mathrm{u}} < 0]$ in this area.

Summary of the steady-state analysis (2)

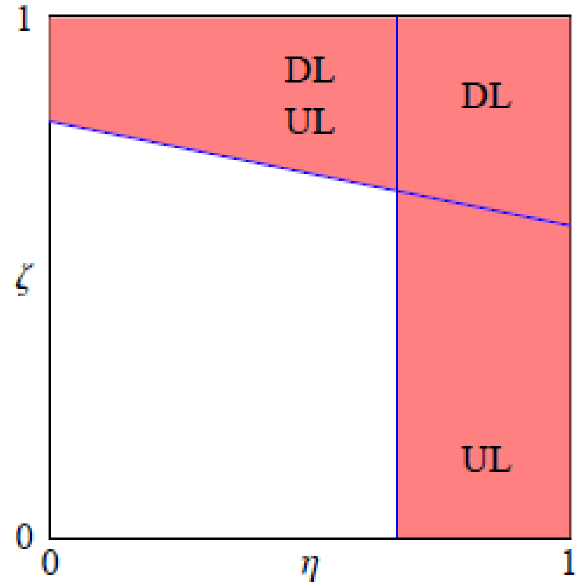

Fig. 2. Solution areas, where DL [UL] refers to a positive download [upload] constrained solution. The horizontal bordering line satisfies $\frac{1}{\mu} = \frac{\eta}{c} + \zeta(z - \frac{1}{c}) + \frac{k}{\lambda}$ and the vertical bordering line satisfies $\eta = \frac{1}{z}(\frac{1}{\mu} - \frac{k}{\lambda})$.

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Steady-state synthesis (1)

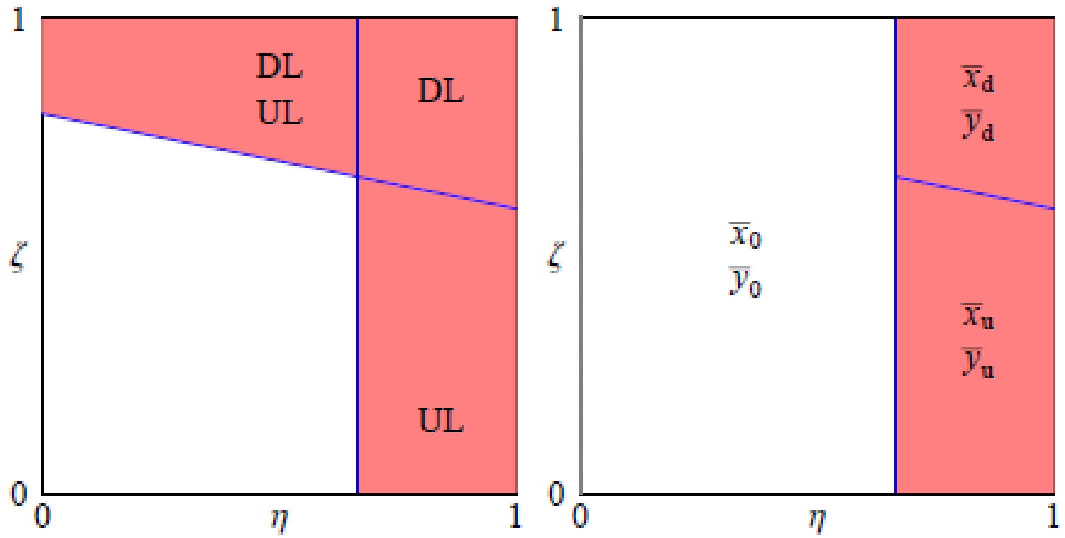

Fig. 1. *Left panel*: Solution areas, where DL [UL] refers to a positive download [upload] constrained solution. The horizontal bordering line satisfies $\frac{1}{\mu} = \frac{\eta}{c} + \zeta(z - \frac{1}{c}) + \frac{k}{\lambda}$ and the vertical bordering line satisfies $\eta = \frac{1}{z}(\frac{1}{\mu} - \frac{k}{\lambda})$. *Right panel*: Steady state synthesis.

Steady-state synthesis (2)

- If

$$\eta < \frac{1}{z}\left(\frac{1}{\mu} - \frac{k}{\lambda}\right)$$

transfer rate < playback rate,
i.e. playback quality problems

- Number of leechers and seeds well estimated by (x_0, y_0):

$$\begin{cases} \bar{x}_0 = x_u|_{\zeta=0} = \frac{\lambda}{\eta}(\frac{1}{\mu} - \frac{k}{\lambda}), \\ \bar{y}_0 = y_u|_{\zeta=0} = 0. \end{cases}$$

- If

$$\eta > \frac{1}{z}\left(\frac{1}{\mu} - \frac{k}{\lambda}\right)$$

transfer rate > playback rate,
i.e. sufficient playback quality

- If further

$$\frac{1}{\mu} \leq \frac{\eta}{c} + \zeta\lambda\left(z - \frac{1}{c}\right) + \frac{k}{\lambda}.$$

DL constrained system (x_d, y_d)

- Otherwise
UL constrained system (x_u, y_u)

Deterministic model vs. stochastic and BitTorrent simulations

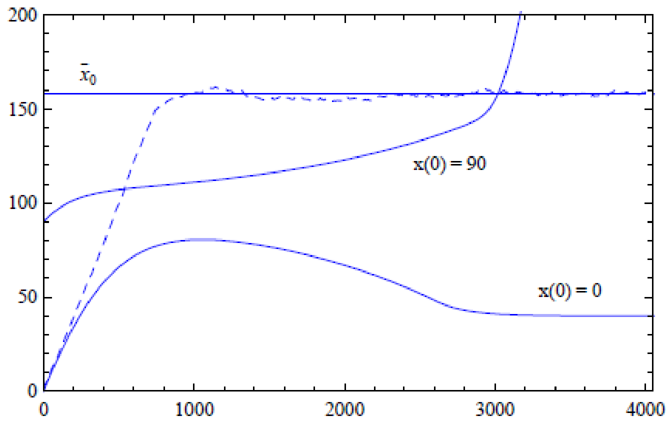

Fig. 5. Comparison of the fluid model (solid smooth lines) against the stochastic model (dashed line) with $\eta = 0.5$ and $\zeta = 0.8$.

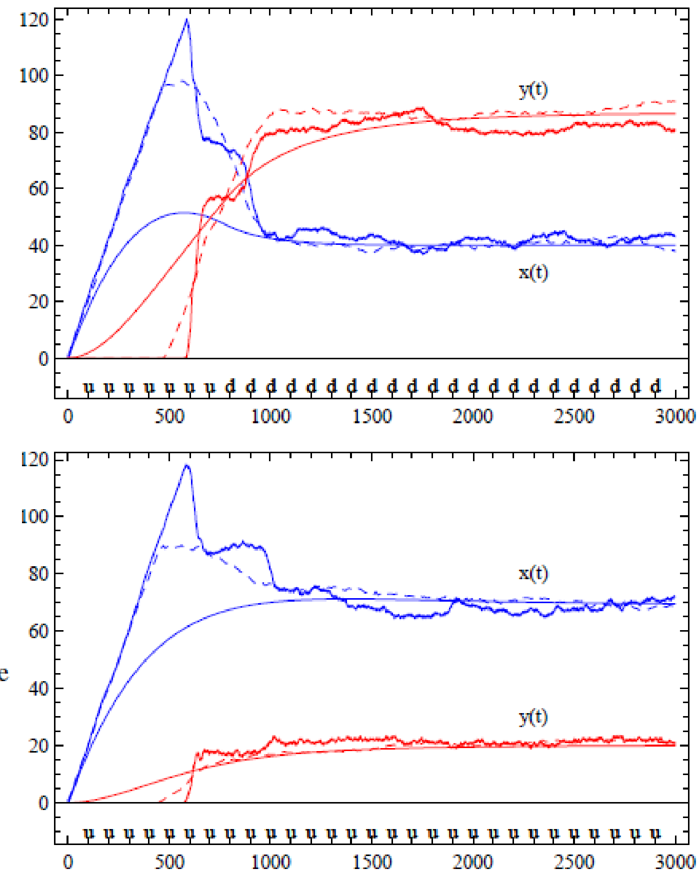

Fig. 4. Comparison of the fluid model (solid smooth line) against the stochastic model (dashed line) and the BitTorrent simulation (solid jagged line) with $\zeta = 0.9$ (upper panel) and $\zeta = 0.3$ (lower panel).

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Conclusions from the P2P VoD model

- Scalability
  - System scalable in the whole parameter space by the synthesis, in particular for any $\eta > 0$
- Stability
  - Consequently, system stable for any $\lambda > 0$
- Performance
  - Playback quality problems if the efficiency parameter η is too small
  - On the other hand, performance even ”scales” (= good quality for all λ) if the efficiency parameter η is sufficiently large
  - Transfer rates for DL and UL constrained cases:

$$\begin{cases} R_d = c \\ 1/z < R_u < c \end{cases}$$

References

[1] D. Qiu and R. Srikant, Modeling and performance analysis of BitTorrent like peer-to-peer networks, in *ACM SIGCOMM*, pp. 367-378, 2004.

[2] D. Qiu and W. Sang, Global stability of peer-to-peer file sharing systems, *Computer Communications*, 31, 2, 212-219, 2008.

[3] S. Aalto, P. Lassila, N. Raatikainen, P. Savolainen, and S. Tarkoma, P2P Video-on-Demand: Steady state and scalability, to appear in *IEEE Globecom*, 2010.